

Supplemental Materials

S.1. Performance of CPMs with an Automatic Link Function Selection Procedure

With CPMs, the primary assumptions are made through the choice of link function. The goodness-of-link test proposed by Genter & Farewell suggests using the log likelihood to discriminate the model fit using the probit, loglog, and cloglog link functions [1]. In a real application when the link function cannot be pre-specified based on preliminary scientific knowledge, one would hope to automate the link function selection. We conducted simulations to investigate the performance of CPMs with an automatic link function selection procedure, in which three CPMs are fitted separately using the probit, loglog, and cloglog links, and the model with the largest log likelihood is selected. We generated data from $Y = H(\beta X + \epsilon)$, where $X \sim N(0, 1)$, $\beta = 0$ under the null hypothesis H_0 , and $\beta = 0.1$ under the alternative hypothesis H_1 . For simplicity, we set $H(y) = y$, that is, no transformation was needed. The error term ϵ was generated from: (a) the standard normal distribution, (b) the standard logistic distribution, (c) the Type I extreme value distribution, and (d) the Type II extreme value distribution. The proper link functions in these scenarios are probit, logit, cloglog, and loglog, respectively. Table S.1 reports the type I error rate and power of CPMs with such a link function selection procedure compared with those with a pre-specified link function. For models with the link function selection procedure, we also report the proportion of times that the chosen link functions were probit, loglog, and cloglog during the 10,000 simulation replicates. Simulations were repeated for sample sizes of 25, 50, 100, 200, 500, and 1000, respectively.

The link function selection based on the log likelihood seems not to perform well in our simulation setting. It generally favors the skewed link functions (cloglog and loglog) under the null no matter what the true error distribution was. Under the alternative hypothesis, it still had difficulty choosing the proper link function, e.g., the proper link function was only favored with large sample sizes, e.g., $n = 1000$. CPMs with such a link function selection procedure have inflated type I error rates. It is interesting to note that with moderate or large sample sizes, e.g., $n \geq 50$, the type I error rates are close to 5% for any pre-specified link function even for those link functions that do not correspond to the true error distributions. This is because there is only one covariate in our simulation setting and under the null hypothesis ($\beta = 0$), there is always a transformation $H^{-1}(\cdot)$ such that $H^{-1}(Y)$ has the distribution the link function specifies [2]. That is, in our simulation setting there is no link function misspecification under the null. However, this is generally not the case when there are multiple covariates in the model.

S.2. Details of Simulations

S.2.1. Estimation with Proper Link Function Specification

In this section, we provide details for simulations with proper link function specification as described in Section 3.1. All results in this section are based on 10,000 simulation replicates for each sample size.

Figure S.1 and Figure S.2 summarize the performance of CPM on estimating conditional means and conditional quantiles using the properly specified CPMs with normal error distribution and with extreme value Type I error distribution, respectively. Percent biases for point estimates, for standard error estimates, and coverage probabilities of the 95% confidence intervals are plotted. Numerical summary for these results and for Figures 6 – 8 in Section 3.1 are provided in Tables S.2 – S.5.

S.2.2. Estimation with Link Function Misspecification In this section, we provide details for simulations with correct and incorrect link function specification as described in Section 3.2. Figures S.3 and S.4 plot the probability density functions (PDFs) of the error distributions (a) - (h) and show the extent of violation to the parallel assumption when probit, logit, cloglog, and loglog link functions are used, respectively. Tables S.6 - S.17 provide detailed summary for mean of point estimates, coverage probabilities of 95% confidence intervals, and relative efficiency (compared to properly specified linear regression for conditional mean and compared to properly specified median regression for conditional median) of CPMs with link function misspecification for $n = 50$, $n = 100$, and $n = 200$. All results are based on 10,000 simulation replicates.

S.2.3. Simulations with More Extreme Covariate Distributions and with an Interaction Term As one of the reviewers suggested, we conducted additional simulations to study the performance of CPMs with more extreme distributions of covariates, with the inclusion of an interaction term between X_1 and X_2 , and with different choices of β . Specifically, we set $X_1 \sim \text{Bernoulli}(0.8)$, $X_2 \sim \text{uniform}(-3, 3)$, and $X_3 = X_1 X_2$. We generated Y from $Y = H(\beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon)$. For simplicity, we set $H(y) = y$, that is, no transformation is needed. We also changed our choice of β , specifically, we set $\beta_1 = 1$, $\beta_2 = 1$, and $\beta_3 = 0.5$.

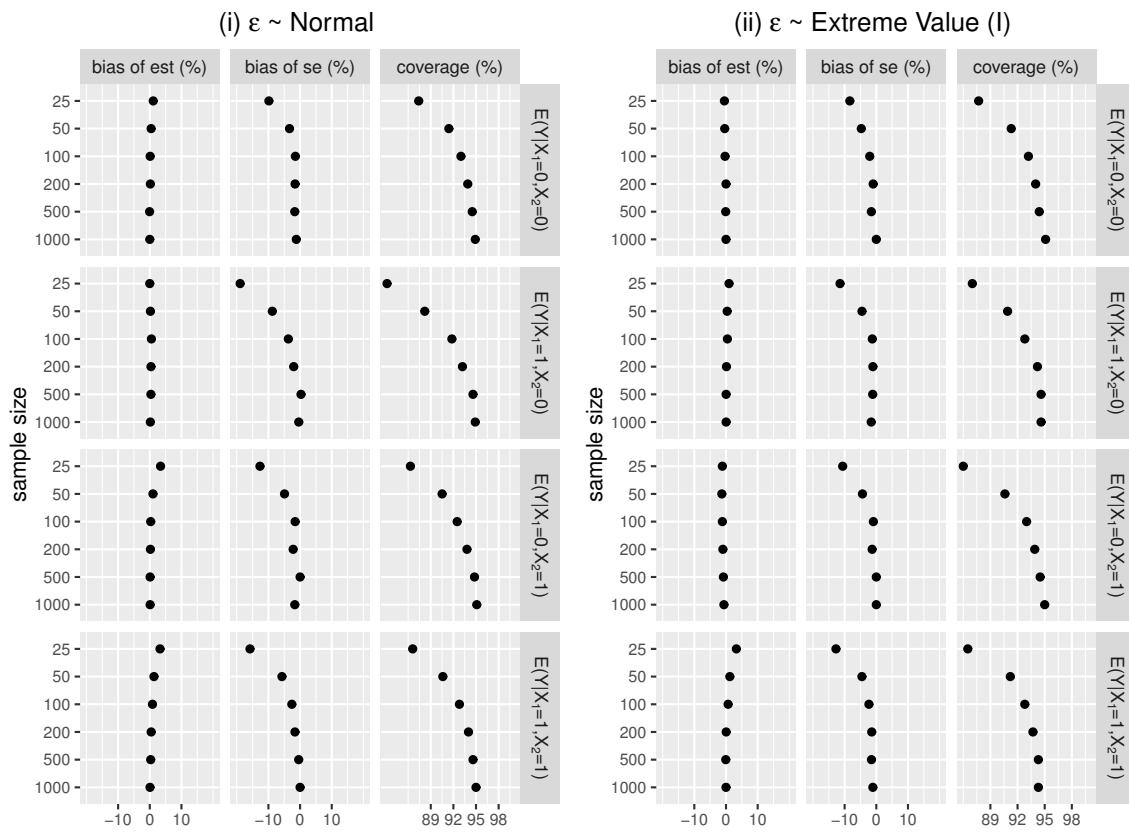


Figure S.1. The performance of CPMs on estimating conditional means with properly specified link functions: (i) $\epsilon \sim \text{Normal}$ and (ii) $\epsilon \sim \text{Extreme Value (I)}$.

Similarly as we did in Section 3.1, we first evaluate the finite sample performance of CPMs with proper link function specification with two error distributions: (i) $\epsilon \sim N(0, 1)$ and (ii) ϵ generated from the extreme value distribution (type I) with location parameter 0 and scale parameter 1.

Figure S.5 summarizes the performance for estimating the regression coefficients, including slopes β_1 , β_2 , β_3 , and intercepts $\alpha(y)$ at $y_1 = -2.707$, $y_2 = -1.327$, $y_3 = 0.729$, $y_4 = 2.865$, and $y_5 = 4.464$. At those values of $\alpha(y)$, the marginal cumulative probabilities of Y with the error distribution (i) are close to 0.1, 0.25, 0.5, 0.75, 0.9, and are close to 0.158, 0.320, 0.567, 0.802, 0.930 with the error distribution (ii), respectively. With this choice of more extreme distribution of X_1 and the inclusion of an interaction term, we find that larger sample sizes are required for CPM to have stable performance. For example, with sample size of 25, the `orm` function failed to converge with 12 iterations (the default setting) for a few cases among the 10,000 simulation replicates (265 cases for error distribution (i) and 250 cases for error distribution (ii), respectively). In addition, we also observed some unusual large standard error estimates (16 cases with at least one standard error estimate greater than 100 for error distribution (i) and 103 cases for error distribution (ii), respectively). Even after removing these cases, we observed relatively large biases in both point estimates of regression coefficients (up to 78%) and their standard error estimates (up to -95%). However, the bias decreases quickly with increasing sample size. For example, in our two simulation settings, the bias for regression coefficients were less than 12% with the sample size of 100, and less than 6% with the sample size of 200. When the sample size gets to 1,000, the bias is less than 2%. Detailed results of this set of simulations is summarized in Table S.18.

We then evaluated the performance of CPMs with link function misspecification similarly as we did in Section 3.2. Figure S.6 and Figure S.7 summarize the performance for estimating conditional means and medians for sample size of 100, respectively. More details of these simulation results are reported in Tables S.19 and S.20, and Tables S.21 and S.22, respectively. The results are generally similar with what we report in Section 3.2: the CPMs with properly specified link functions have good performance for estimating conditional means and medians in terms of bias and coverage probability of 95% confidence intervals; the CPMs seem to have reasonable performance with

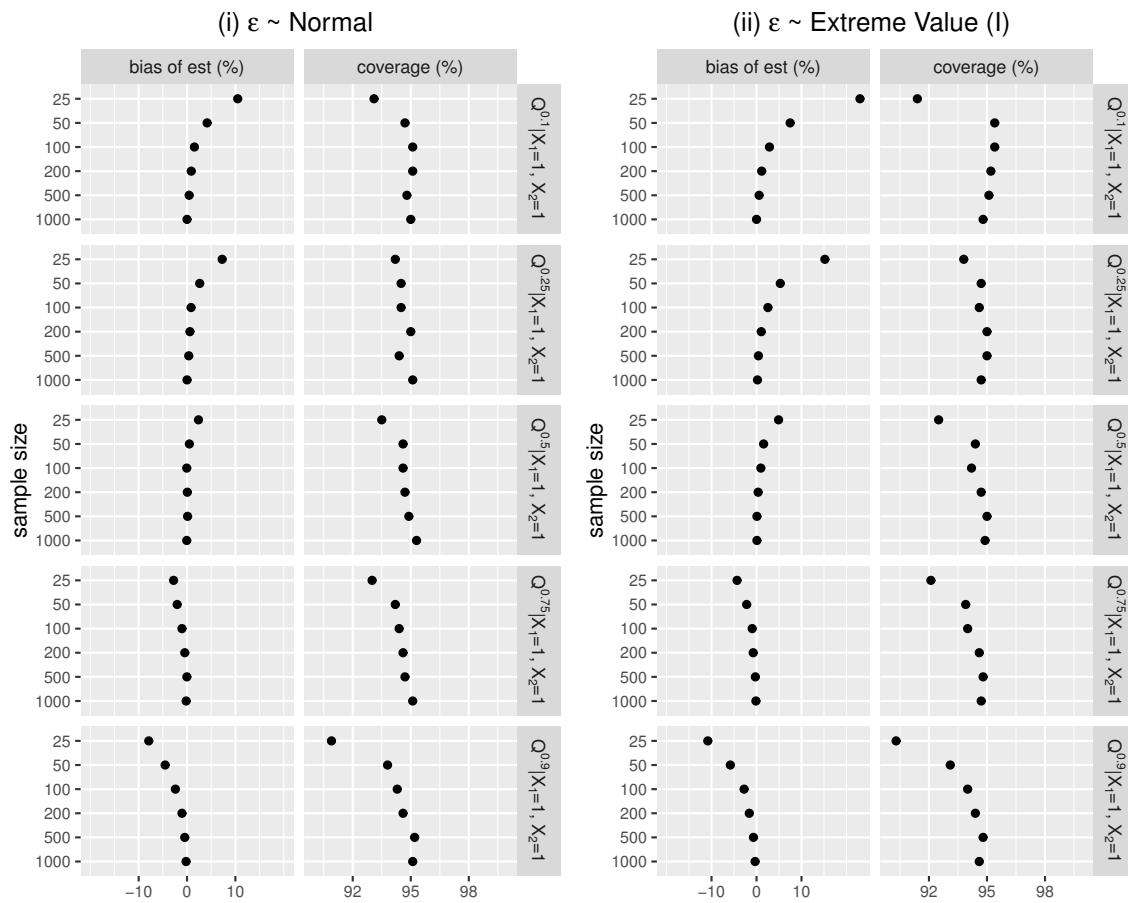


Figure S.2. The performance of CPMs on estimating conditional 10^{th} , 25^{th} , 50^{th} , 75^{th} , and 90^{th} quantiles with properly specified link functions: (i) $\epsilon \sim \text{Normal}$ and (ii) $\epsilon \sim \text{Extreme Value (I)}$.

minor or moderate link function misspecification; however, poor performance was observed with severe link function missppecification. Also, we observed that with this choice of more extreme distribution of X_1 and the inclusion of an interaction term, CPMs are generally less efficient for estimating the conditional means and medians compared with properly specified least square regression models and median regression models, i.e., MSE ratios are generally less than 1.

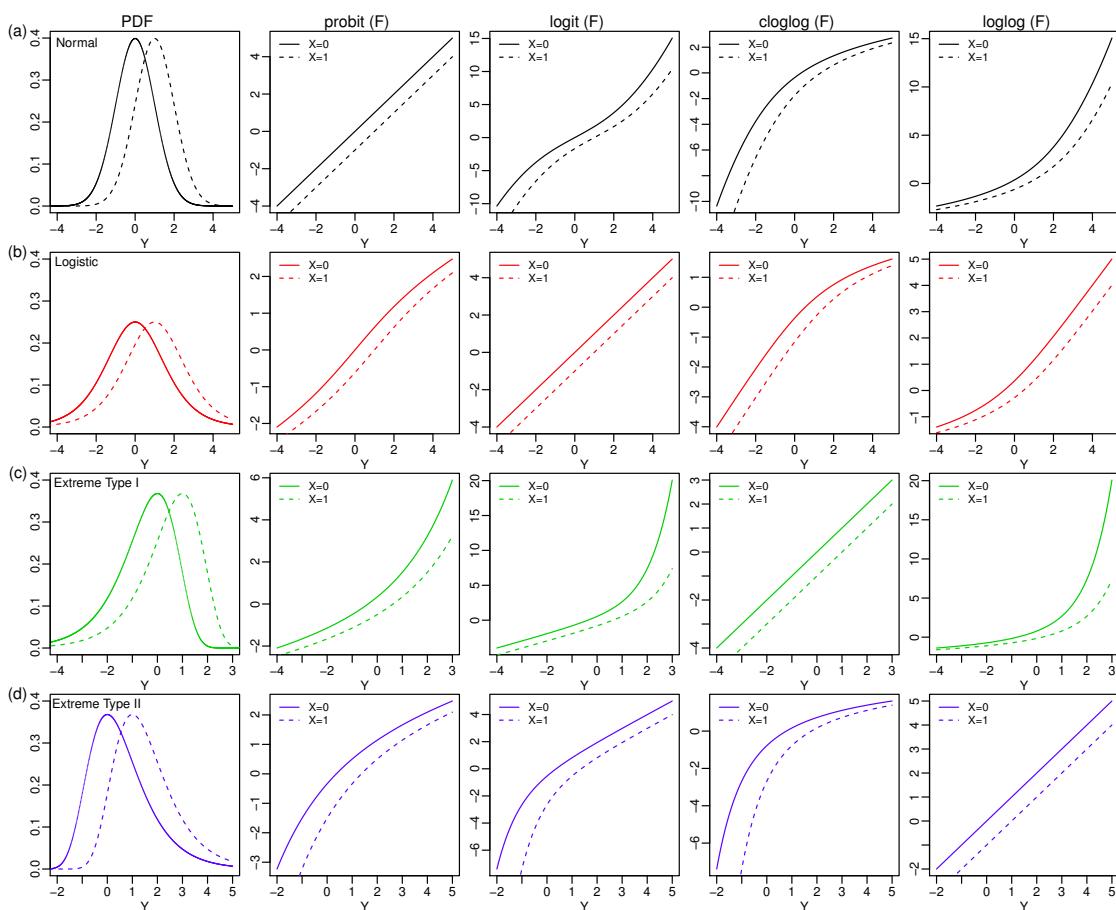


Figure S.3. The probability density functions of error distributions for (a) – (d) and the extent of violation to the parallel assumption with commonly used link functions.

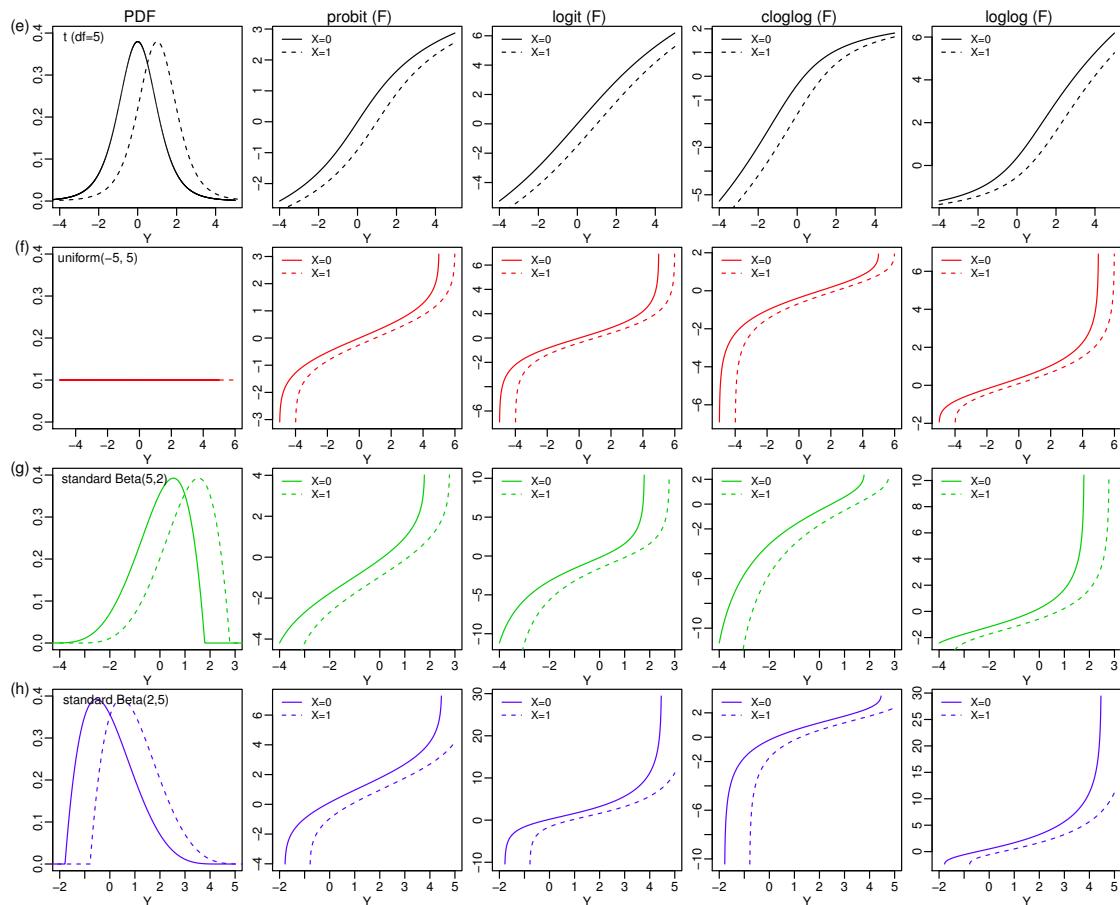


Figure S.4. The probability density functions of error distributions for (e) – (h) and the extent of violation to the parallel assumption with commonly used link functions.

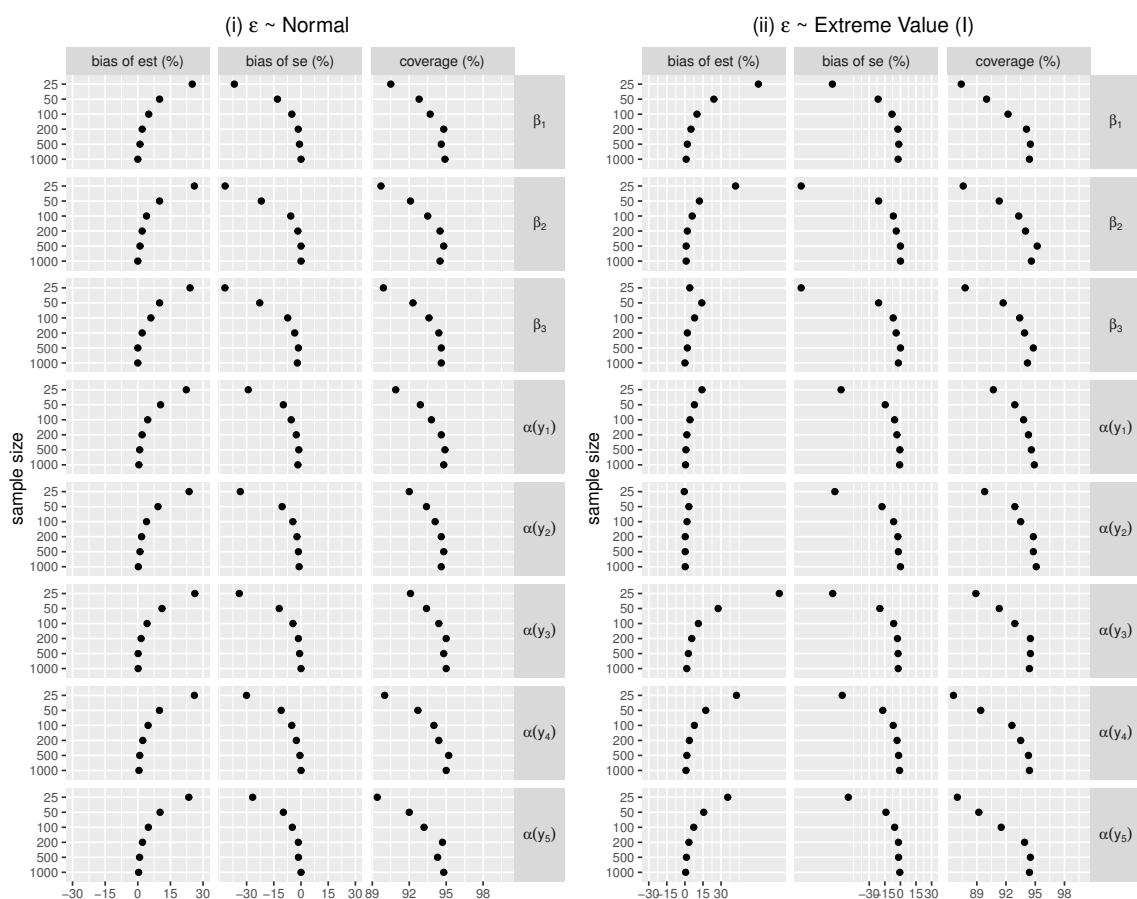


Figure S.5. The performance of CPMs on estimating regression coefficients under more extreme covariates distributions and an interaction term with properly specified link functions.

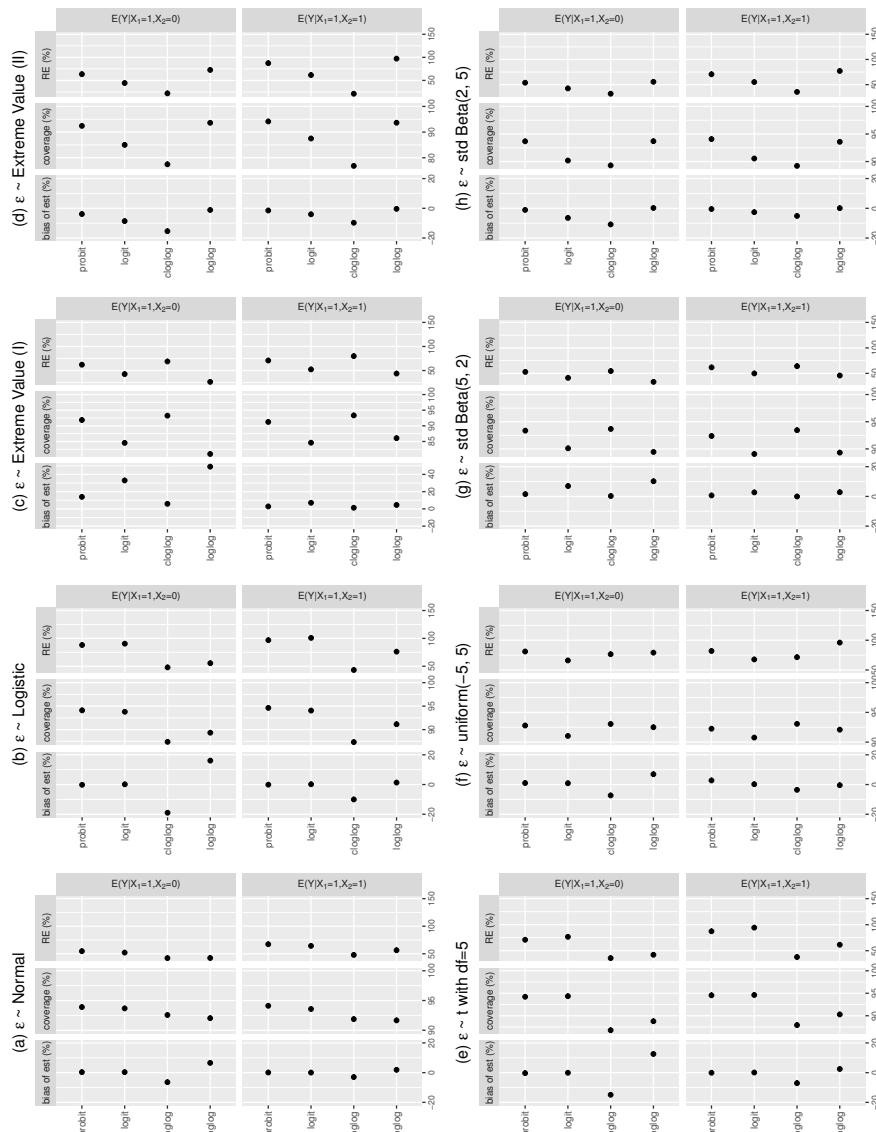


Figure S.6. The performance of CPMs on estimating conditional means under more extreme covariates distributions and an interaction term with commonly used link functions.

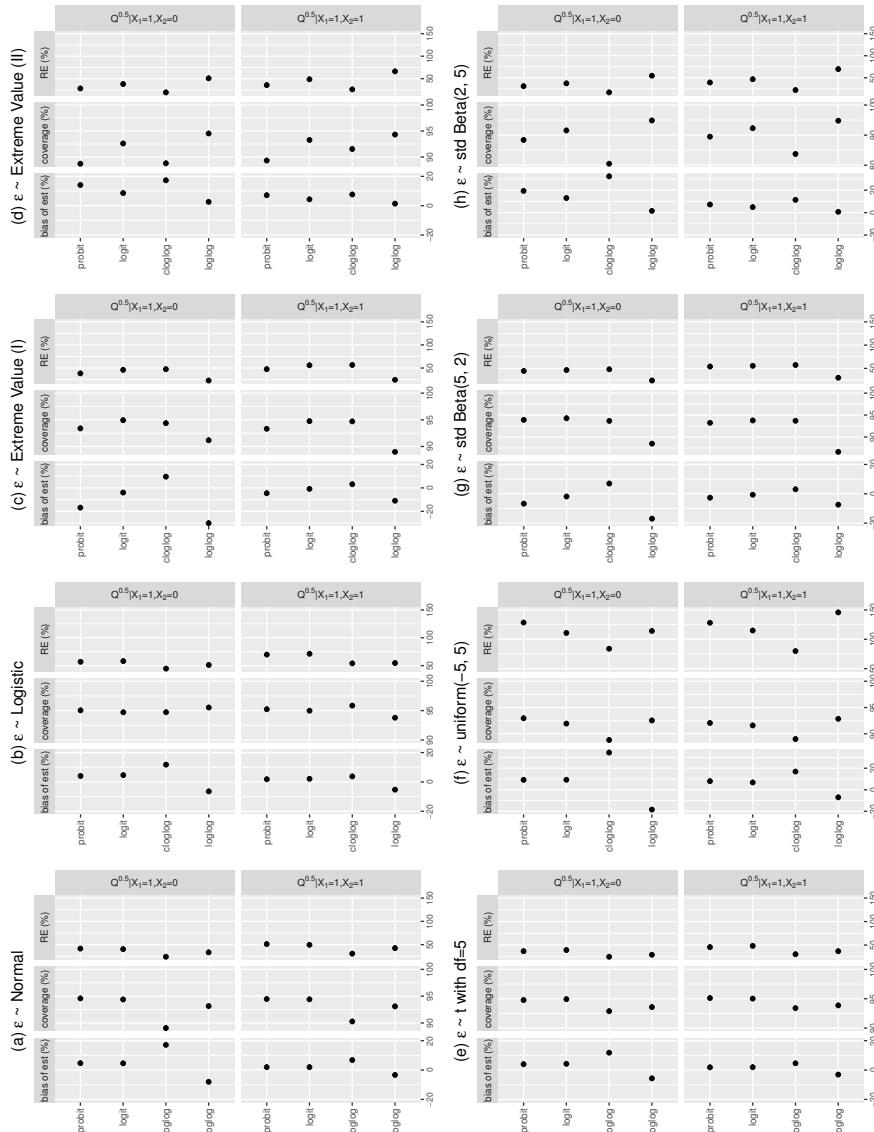


Figure S.7. The performance of CPMs on estimating conditional medians under more extreme covariates distributions and an interaction term with commonly used link functions.

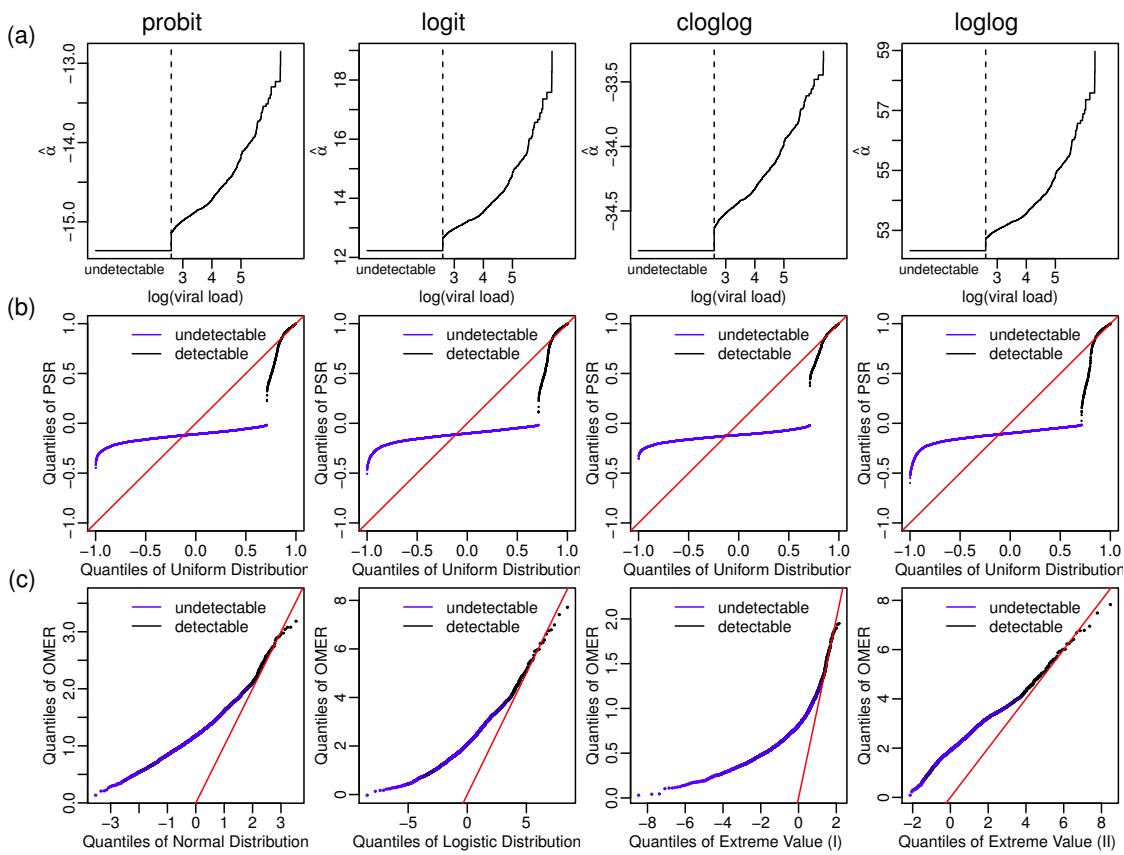


Figure S.8. (a): the estimated intercepts $\hat{\alpha}(y)$ resulting from the CPMs using the probit, logit, cloglog, and loglog link functions, which can be interpreted as semiparametric estimates of the best transformation for the 6-month viral load. (b): QQ-plots of probability-scale residuals (PSRs). (c): QQ-plots of observed-minus-expected residuals (OMERs), removing the residual for the observation with the largest value of viral load.

S.2.4. Computation Time In this section, we provide details of simulations for computation time after binning continuous outcomes based on quantiles as described in Section 3.3. Table S.23 and Table S.24 summarize the average estimation time, average number of distinct outcomes, percent bias, and root mean squared error(rMSE) with and without binning for data generated from normal and Type I extreme value distributions.

S.3. Application Examples

Table S.25 summarizes the descriptive statistics of variables in the application example of Section 4. Figure S.8 displays the estimated transformations and the QQ-plots of PSRs and OMERs. Figure S.9 plots the PSRs vs. age from CPMs including and not including age.

S.4. Simulations of CPMs for Measurements Subject to Detection Limits

We conducted simulations to investigate the performance of CPMs for handling measurements subject to detection limits. We generated data with sample size of 100 from $Y^* = \alpha + \beta X + \epsilon$, where $X \sim N(0, 1)$, $\epsilon \sim N(0, 1)$, $\alpha = 3$, $\beta = 0$ under the null hypothesis (H_0), and $\beta = 0.25$ under the alternative hypothesis (H_1). The outcome variable Y was then generated by left censoring Y^* at the detection limit (DL). That is, we set Y as undetectable if $Y^* < \text{DL}$ and set $Y = Y^*$ if $Y^* \geq \text{DL}$. We changed the values of DL to vary the proportion of undetectable measurements. Specifically, we set DL as 1.64, 2.28, 3.0, 3.72, and 4.36 so that the marginal proportion of undetectable measurements were 10%, 25%, 50%, 75%, and 90%, respectively.

We fitted the properly specified CPM using the probit link function. For purpose of comparison, we also fitted the logistic regression using dichotomized outcomes (undetectable vs. detectable) and two separate linear regression models: one imputing values below the detection limit as DL and the other imputing values below the detection limit as 0. To compare results with the logistic regression models, we also fitted CPMs with the logit link function.

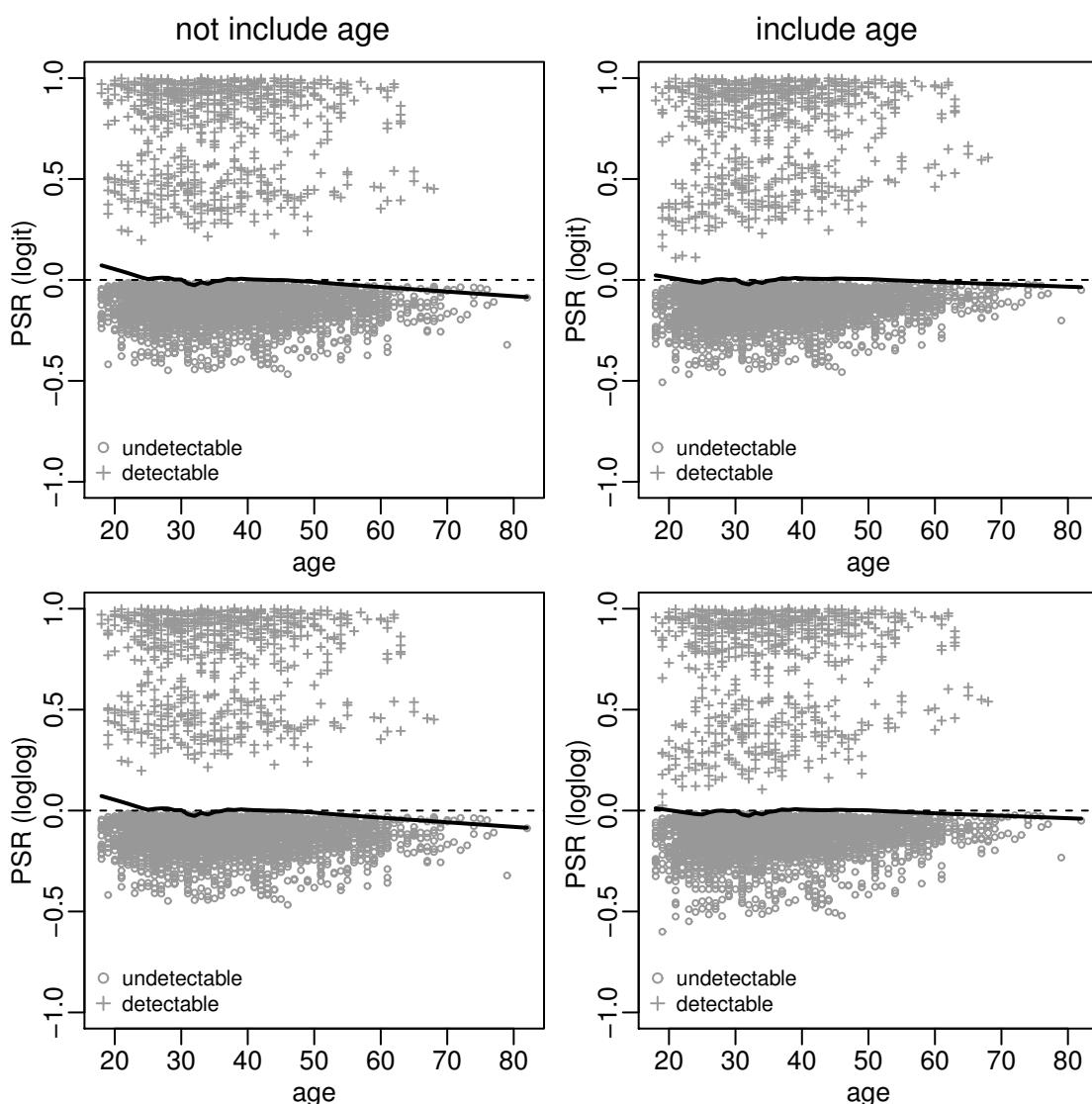


Figure S.9. Residual-by-predictor plots using PSRs from CPMs using the logit link function (top panel) and the loglog link function (bottom panel). Smoothed curves using Friedman's super smoother are added.

Table S.26 summarizes the type I error rate and power of these approaches under various proportions of undetectable measurements. We found that the performance of CPMs with the probit and logit link functions were generally similar. Type I error rates were conserved and CPMs were more efficient than logistic regression using dichotomized viral loads, although the gains in efficiency decreased as the proportion of undetectable measures increased. CPMs were also slightly more efficient than linear regression with undetectable viral load imputed.

References

1. Genter FC, Farewell VT. Goodness-of-link testing in ordinal regression models. *Canadian Journal of Statistics* 1985; **13**(1):37–44.
2. Zeng D, Lin D. Maximum likelihood estimation in semiparametric regression models with censored data. *Journal of the Royal Statistical Society. Series B (Methodological)* 2007; **69**(4):507–564.

Table S.1. Type I error rate and power of CPMs with an automatic link function selection procedure (selecting the link function with the highest likelihood among probit, cloglog, and loglog) compared with those with a pre-specified link function. The numbers in the parentheses are the proportions of chosen link function being probit, cloglog, or loglog during 10,000 simulation replicates.

True error distribution		link function selection	probit	cloglog	loglog
Normal					
	$n = 25$	H_0 0.118 (9.78%, 45.02%, 45.20%) H_1 0.153 (10.86%, 44.67%, 44.47%)	0.068 0.096	0.068 0.098	0.074 0.095
	$n = 50$	H_0 0.102 (12.52%, 43.51%, 43.97%) H_1 0.176 (13.95%, 42.89%, 43.16%)	0.057 0.113	0.060 0.106	0.060 0.112
	$n = 100$	H_0 0.099 (13.20%, 44.04%, 42.76%) H_1 0.247 (18.54%, 40.91%, 40.55%)	0.054 0.173	0.058 0.156	0.057 0.161
	$n = 200$	H_0 0.096 (14.83%, 43.31%, 41.86%) H_1 0.373 (24.16%, 37.31%, 38.53%)	0.052 0.294	0.053 0.261	0.056 0.251
	$n = 500$	H_0 0.091 (15.10%, 41.58%, 43.32%) H_1 0.678 (37.19%, 31.02%, 31.79%)	0.048 0.607	0.051 0.522	0.055 0.526
	$n = 1000$	H_0 0.099 (15.62%, 42.44%, 41.94%) H_1 0.910 (50.02%, 24.51%, 25.47%)	0.058 0.880	0.057 0.808	0.055 0.814
Logistic					
	$n = 25$	H_0 0.116 (10.15%, 45.87%, 43.98%) H_1 0.126 (10.75%, 45.48%, 43.77%)	0.066 0.076	0.072 0.080	0.069 0.078
	$n = 50$	H_0 0.106 (12.02%, 43.35%, 44.63%) H_1 0.129 (13.37%, 42.53%, 44.10%)	0.057 0.077	0.064 0.077	0.061 0.077
	$n = 100$	H_0 0.100 (13.15%, 42.90%, 43.95%) H_1 0.142 (15.73%, 41.76%, 42.51%)	0.054 0.089	0.056 0.083	0.057 0.087
	$n = 200$	H_0 0.099 (14.78%, 42.44%, 42.78%) H_1 0.194 (19.45%, 40.16%, 40.39%)	0.054 0.137	0.055 0.121	0.055 0.119
	$n = 500$	H_0 0.091 (14.87%, 42.83%, 42.30%) H_1 0.317 (26.45%, 36.87%, 36.68%)	0.050 0.244	0.050 0.205	0.050 0.204
	$n = 1000$	H_0 0.095 (15.60%, 42.45%, 41.95%) H_1 0.506 (35.05%, 32.52%, 32.43%)	0.052 0.428	0.054 0.350	0.051 0.358
Extreme Type I					
	$n = 25$	H_0 0.116 (10.15%, 45.87%, 43.98%) H_1 0.147 (10.32%, 47.74%, 41.94%)	0.066 0.089	0.072 0.100	0.069 0.085
	$n = 50$	H_0 0.106 (12.02%, 43.35%, 44.63%) H_1 0.165 (13.41%, 47.42%, 39.17%)	0.057 0.104	0.064 0.119	0.061 0.088
	$n = 100$	H_0 0.100 (13.15%, 42.90%, 43.95%) H_1 0.227 (16.23%, 50.30%, 33.47%)	0.054 0.152	0.056 0.171	0.057 0.109
	$n = 200$	H_0 0.099 (14.78%, 42.44%, 42.78%) H_1 0.352 (19.37%, 55.72%, 24.91%)	0.054 0.256	0.055 0.296	0.055 0.166
	$n = 500$	H_0 0.091 (14.87%, 42.83%, 42.30%) H_1 0.652 (21.80%, 66.99%, 11.21%)	0.050 0.525	0.050 0.606	0.050 0.312
	$n = 1000$	H_0 0.095 (15.60%, 42.45%, 41.95%) H_1 0.903 (21.17%, 75.20%, 3.63%)	0.052 0.819	0.054 0.886	0.051 0.534
Extreme Type II					
	$n = 25$	H_0 0.116 (10.15%, 45.87%, 43.98%) H_1 0.145 (10.72%, 43.29%, 45.99%)	0.066 0.088	0.072 0.086	0.069 0.096
	$n = 50$	H_0 0.106 (12.02%, 43.35%, 44.63%) H_1 0.166 (13.71%, 38.10%, 48.19%)	0.057 0.106	0.064 0.088	0.061 0.118
	$n = 100$	H_0 0.100 (13.15%, 42.90%, 43.95%) H_1 0.224 (15.54%, 32.69%, 51.77%)	0.054 0.148	0.056 0.103	0.057 0.174
	$n = 200$	H_0 0.099 (14.78%, 42.44%, 42.78%) H_1 0.356 (18.58%, 23.89%, 57.53%)	0.054 0.254	0.055 0.163	0.055 0.305
	$n = 500$	H_0 0.091 (14.87%, 42.83%, 42.30%) H_1 0.652 (21.52%, 11.24%, 67.24%)	0.050 0.524	0.050 0.311	0.050 0.609
	$n = 1000$	H_0 0.095 (15.60%, 42.45%, 41.95%) H_1 0.900 (21.07%, 3.72%, 75.21%)	0.052 0.818	0.054 0.534	0.051 0.882

Table S.2. The performance of CPMs on estimating the slopes β_1 and β_2 and the intercepts at $y_1 = 0.368$, $y_2 = 0.719$, $y_3 = 1.649$, $y_4 = 3.781$, and $y_5 = 7.389$ with properly specified link functions. These results are also summarized in Figure 6.

	true	(i) $\epsilon \sim \text{Normal}$				(ii) $\epsilon \sim \text{Extreme Type I}$			
		est	est.se	emp.se	CP	est	est.se	emp.se	CP
<i>n</i> = 25									
	β_1	1.00	1.13	0.472	0.550	0.922	1.19	0.515	0.610
	β_2	-0.50	-0.57	0.242	0.278	0.926	-0.60	0.269	0.314
	$\alpha(y_1)$	-1.00	-1.07	0.436	0.425	0.969	-1.09	0.490	0.532
	$\alpha(y_2)$	-0.33	-0.36	0.372	0.410	0.943	-0.34	0.395	0.434
	$\alpha(y_3)$	0.50	0.56	0.376	0.423	0.940	0.60	0.368	0.435
	$\alpha(y_4)$	1.33	1.49	0.439	0.504	0.936	1.58	0.454	0.526
	$\alpha(y_5)$	2.00	2.22	0.533	0.582	0.948	2.27	0.573	0.625
<i>n</i> = 50									
	β_1	1.00	1.06	0.314	0.331	0.941	1.08	0.334	0.361
	β_2	-0.50	-0.53	0.160	0.171	0.940	-0.54	0.172	0.187
	$\alpha(y_1)$	-1.00	-1.06	0.298	0.311	0.957	-1.04	0.327	0.343
	$\alpha(y_2)$	-0.33	-0.35	0.251	0.261	0.949	-0.33	0.266	0.281
	$\alpha(y_3)$	0.50	0.53	0.251	0.262	0.945	0.55	0.243	0.262
	$\alpha(y_4)$	1.33	1.40	0.289	0.303	0.945	1.44	0.287	0.311
	$\alpha(y_5)$	2.00	2.12	0.354	0.375	0.948	2.16	0.379	0.390
<i>n</i> = 100									
	β_1	1.00	1.03	0.217	0.222	0.947	1.04	0.228	0.235
	β_2	-0.50	-0.52	0.109	0.114	0.943	-0.52	0.116	0.121
	$\alpha(y_1)$	-1.00	-1.03	0.203	0.210	0.948	-1.02	0.226	0.231
	$\alpha(y_2)$	-0.33	-0.33	0.174	0.178	0.948	-0.33	0.184	0.188
	$\alpha(y_3)$	0.50	0.52	0.174	0.176	0.949	0.52	0.167	0.173
	$\alpha(y_4)$	1.33	1.36	0.200	0.205	0.947	1.38	0.193	0.201
	$\alpha(y_5)$	2.00	2.05	0.241	0.248	0.947	2.09	0.255	0.264
<i>n</i> = 200									
	β_1	1.00	1.01	0.152	0.155	0.946	1.02	0.158	0.162
	β_2	-0.50	-0.51	0.076	0.079	0.942	-0.51	0.080	0.081
	$\alpha(y_1)$	-1.00	-1.01	0.142	0.145	0.951	-1.01	0.158	0.159
	$\alpha(y_2)$	-0.33	-0.33	0.122	0.123	0.950	-0.33	0.129	0.131
	$\alpha(y_3)$	0.50	0.51	0.122	0.125	0.947	0.51	0.117	0.118
	$\alpha(y_4)$	1.33	1.35	0.140	0.143	0.948	1.35	0.133	0.136
	$\alpha(y_5)$	2.00	2.03	0.168	0.172	0.951	2.04	0.174	0.178
<i>n</i> = 500									
	β_1	1.00	1.01	0.095	0.097	0.946	1.01	0.099	0.100
	β_2	-0.50	-0.50	0.048	0.047	0.951	-0.51	0.050	0.050
	$\alpha(y_1)$	-1.00	-1.01	0.089	0.089	0.949	-1.00	0.099	0.100
	$\alpha(y_2)$	-0.33	-0.33	0.077	0.077	0.950	-0.33	0.081	0.082
	$\alpha(y_3)$	0.50	0.50	0.077	0.078	0.946	0.50	0.073	0.074
	$\alpha(y_4)$	1.33	1.34	0.088	0.089	0.949	1.34	0.083	0.085
	$\alpha(y_5)$	2.00	2.01	0.105	0.106	0.950	2.02	0.108	0.110
<i>n</i> = 1000									
	β_1	1.00	1.00	0.067	0.067	0.951	1.00	0.070	0.070
	β_2	-0.50	-0.50	0.034	0.034	0.948	-0.50	0.035	0.035
	$\alpha(y_1)$	-1.00	-1.00	0.063	0.063	0.949	-1.00	0.070	0.070
	$\alpha(y_2)$	-0.33	-0.33	0.054	0.054	0.953	-0.33	0.057	0.057
	$\alpha(y_3)$	0.50	0.50	0.054	0.054	0.951	0.50	0.052	0.052
	$\alpha(y_4)$	1.33	1.33	0.062	0.062	0.950	1.33	0.059	0.059
	$\alpha(y_5)$	2.00	2.00	0.074	0.075	0.949	2.01	0.076	0.076

est is the mean of the point estimates.

est.se is the mean of the standard error estimates.

emp.se is the standard deviation of the point estimates

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

Table S.3. The performance of CPMs on estimating the conditional CDFs evaluated at $y_1 = 0.368$, $y_2 = 0.719$, $y_3 = 1.649$, $y_4 = 3.781$, and $y_5 = 7.389$. These results are also summarized in Figure 8.

	(i) $\epsilon \sim \text{Normal}$					(ii) $\epsilon \sim \text{Extreme Type I}$					
	true	est	est.se	emp.se	CP	true	est	est.se	emp.se	CP	
$n = 25$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0701	0.0680	0.0620	0.953	0.2000	0.1949	0.1074	0.0936	0.939
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2051	0.1275	0.1134	0.937	0.3534	0.3501	0.1492	0.1296	0.932
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.4997	0.1694	0.1525	0.935	0.6321	0.6326	0.1710	0.1485	0.930
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7964	0.1264	0.1119	0.935	0.8991	0.8973	0.1006	0.0860	0.938
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9304	0.0693	0.0602	0.950	0.9887	0.9825	0.0350	0.0320	0.962
$n = 50$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0679	0.0455	0.0423	0.947	0.2000	0.1980	0.0696	0.0664	0.947
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2036	0.0851	0.0817	0.945	0.3534	0.3515	0.0954	0.0914	0.944
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.5024	0.1148	0.1096	0.946	0.6321	0.6341	0.1115	0.1063	0.945
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7980	0.0849	0.0807	0.944	0.8991	0.8994	0.0676	0.0629	0.942
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9326	0.0453	0.0417	0.945	0.9887	0.9855	0.0203	0.0180	0.957
$n = 100$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0679	0.0315	0.0304	0.947	0.2000	0.1988	0.0477	0.0467	0.950
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2044	0.0598	0.0583	0.944	0.3534	0.3520	0.0658	0.0640	0.945
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.5018	0.0797	0.0778	0.947	0.6321	0.6323	0.0772	0.0747	0.944
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7976	0.0593	0.0575	0.945	0.8991	0.8991	0.0471	0.0455	0.941
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9328	0.0311	0.0300	0.947	0.9887	0.9872	0.0128	0.0118	0.950
$n = 200$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0672	0.0219	0.0216	0.948	0.2000	0.1994	0.0331	0.0330	0.950
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2034	0.0416	0.0412	0.950	0.3534	0.3529	0.0455	0.0450	0.952
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.5004	0.0555	0.0550	0.948	0.6321	0.6326	0.0535	0.0525	0.946
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7972	0.0415	0.0409	0.949	0.8991	0.8997	0.0328	0.0324	0.949
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9330	0.0220	0.0214	0.947	0.9887	0.9880	0.0086	0.0082	0.948
$n = 500$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0667	0.0139	0.0136	0.950	0.2000	0.1998	0.0210	0.0208	0.948
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2029	0.0266	0.0261	0.945	0.3534	0.3533	0.0287	0.0284	0.950
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.4997	0.0354	0.0348	0.949	0.6321	0.6324	0.0335	0.0331	0.946
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7964	0.0262	0.0259	0.949	0.8991	0.8995	0.0208	0.0206	0.948
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9332	0.0135	0.0136	0.950	0.9887	0.9885	0.0053	0.0052	0.947
$n = 1000$											
	$F(y_1 X_1 = 1, X_2 = 1)$	0.0668	0.0669	0.0097	0.0097	0.949	0.2000	0.1998	0.0149	0.0147	0.948
	$F(y_2 X_1 = 1, X_2 = 1)$	0.2033	0.2035	0.0184	0.0185	0.946	0.3534	0.3534	0.0202	0.0201	0.949
	$F(y_3 X_1 = 1, X_2 = 1)$	0.5000	0.5002	0.0243	0.0246	0.952	0.6321	0.6321	0.0236	0.0234	0.947
	$F(y_4 X_1 = 1, X_2 = 1)$	0.7967	0.7969	0.0182	0.0183	0.953	0.8991	0.8991	0.0149	0.0146	0.945
	$F(y_5 X_1 = 1, X_2 = 1)$	0.9332	0.9331	0.0097	0.0096	0.948	0.9887	0.9885	0.0038	0.0037	0.950

est is the mean of the point estimates.

est.se is the mean of the standard error estimates.

emp.se is the standard deviation of the point estimates

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

Table S.4. The performance of CPMs on estimating conditional means. These results are also summarized in Figure S.1.

		(i) $\epsilon \sim \text{Normal}$					(ii) $\epsilon \sim \text{Extreme Type I}$					
		true	est	est.se	emp.se	CP	true	est	est.se	emp.se	CP	
	$n = 25$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.668	0.531	0.589	0.874	1.00	0.995	0.285	0.311	0.877
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.480	1.358	1.675	0.832	2.72	2.746	0.758	0.856	0.870
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.034	0.393	0.450	0.863	0.61	0.603	0.219	0.245	0.860
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.808	1.043	1.239	0.866	1.65	1.704	0.603	0.691	0.865
	$n = 50$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.657	0.377	0.390	0.914	1.00	0.996	0.201	0.211	0.913
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.489	1.020	1.118	0.882	2.72	2.730	0.549	0.575	0.909
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.010	0.271	0.285	0.905	0.61	0.602	0.153	0.160	0.906
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.756	0.740	0.785	0.906	1.65	1.670	0.421	0.441	0.912
	$n = 100$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.652	0.265	0.269	0.930	1.00	0.997	0.142	0.145	0.932
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.503	0.753	0.782	0.918	2.72	2.732	0.391	0.396	0.928
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.003	0.190	0.193	0.925	0.61	0.603	0.107	0.108	0.930
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.743	0.524	0.538	0.928	1.65	1.661	0.294	0.301	0.928
	$n = 200$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.653	0.187	0.190	0.939	1.00	1.000	0.100	0.101	0.940
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.497	0.535	0.546	0.932	2.72	2.724	0.276	0.279	0.942
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.002	0.134	0.137	0.938	0.61	0.604	0.075	0.076	0.939
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.732	0.366	0.372	0.940	1.65	1.651	0.205	0.208	0.937
	$n = 500$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.649	0.117	0.119	0.945	1.00	0.999	0.063	0.064	0.944
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.496	0.341	0.340	0.946	2.72	2.721	0.174	0.176	0.946
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.001	0.084	0.084	0.948	0.61	0.605	0.048	0.048	0.945
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.728	0.230	0.231	0.946	1.65	1.649	0.129	0.131	0.943
	$n = 1000$											
		$E(Y X_1 = 0, X_2 = 0)$	1.65	1.650	0.083	0.084	0.949	1.00	1.000	0.045	0.045	0.951
		$E(Y X_1 = 1, X_2 = 0)$	4.48	4.488	0.241	0.242	0.949	2.72	2.721	0.123	0.125	0.946
		$E(Y X_1 = 0, X_2 = 1)$	1.00	1.001	0.059	0.060	0.951	0.61	0.606	0.034	0.034	0.950
		$E(Y X_1 = 1, X_2 = 1)$	2.72	2.722	0.162	0.162	0.950	1.65	1.650	0.091	0.092	0.943

est is the mean of the point estimates.

est.se is the mean of the standard error estimates.

emp.se is the standard deviation of the point estimates

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

Table S.5. The performance of CPMs on estimating the conditional 10th, 25th, 50th, 75th, and 90th quantiles. These results are also summarized in Figure S.2.

		(i) $\epsilon \sim \text{Normal}$				(ii) $\epsilon \sim \text{Extreme Type I}$				
		true	est	emp.se	CP	true	est	emp.se	CP	
	$n = 25$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.506	0.2657	0.931	0.174	0.214	0.1639	0.914
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.901	0.4165	0.942	0.474	0.546	0.3156	0.938
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.688	0.7428	0.935	1.143	1.199	0.5686	0.925
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.146	1.4068	0.930	2.286	2.187	0.9519	0.921
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.469	2.6814	0.909	3.796	3.385	1.4476	0.903
	$n = 50$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.477	0.1623	0.947	0.174	0.187	0.0904	0.954
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.862	0.2602	0.945	0.474	0.499	0.1872	0.947
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.657	0.4863	0.946	1.143	1.161	0.3615	0.944
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.170	0.9467	0.942	2.286	2.236	0.6473	0.939
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.670	1.8859	0.938	3.796	3.575	1.0389	0.931
	$n = 100$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.465	0.1083	0.951	0.174	0.179	0.0590	0.954
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.847	0.1773	0.945	0.474	0.486	0.1236	0.946
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.648	0.3305	0.946	1.143	1.154	0.2509	0.942
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.202	0.6600	0.944	2.286	2.264	0.4546	0.940
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.796	1.3193	0.943	3.796	3.691	0.7401	0.940
	$n = 200$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.462	0.0738	0.951	0.174	0.176	0.0409	0.952
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.845	0.1221	0.950	0.474	0.479	0.0839	0.950
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.650	0.2311	0.947	1.143	1.147	0.1725	0.947
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.221	0.4636	0.946	2.286	2.269	0.3215	0.946
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.877	0.9432	0.946	3.796	3.735	0.5288	0.944
	$n = 500$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.460	0.0469	0.948	0.174	0.175	0.0257	0.951
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.843	0.0783	0.944	0.474	0.476	0.0529	0.950
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.651	0.1465	0.949	1.143	1.144	0.1081	0.950
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.235	0.2942	0.947	2.286	2.280	0.2022	0.948
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.911	0.5867	0.952	3.796	3.770	0.3385	0.948
	$n = 1000$									
		$Q^{0.10} X_1 = 1, X_2 = 1$	0.458	0.458	0.0325	0.950	0.174	0.174	0.0181	0.948
		$Q^{0.25} X_1 = 1, X_2 = 1$	0.840	0.840	0.0536	0.951	0.474	0.475	0.0371	0.947
		$Q^{0.50} X_1 = 1, X_2 = 1$	1.649	1.648	0.1009	0.953	1.143	1.144	0.0759	0.949
		$Q^{0.75} X_1 = 1, X_2 = 1$	3.236	3.230	0.2037	0.951	2.286	2.283	0.1432	0.947
		$Q^{0.90} X_1 = 1, X_2 = 1$	5.939	5.927	0.4167	0.951	3.796	3.784	0.2420	0.946

est is the mean of the point estimates.

emp.se is the standard deviation of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

Table S.6. The performance of CPMs on estimating conditional means with the sample size of 50. The results are based on 10,000 simulation replicates.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.938	0.979	0.01	0.931	0.950	0.02	0.940
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.934	0.978	0.99	0.930	0.956	0.92	0.924
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.50	0.930	0.967	-0.48	0.924	0.925	-0.38	0.857
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.934	0.962	0.49	0.928	0.944	0.44	0.908
(b) $\epsilon \sim \text{Logistic}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.937	1.012	0.00	0.938	1.043	0.04	0.912
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.936	1.014	1.00	0.935	1.049	0.90	0.932
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.51	0.938	0.982	-0.50	0.936	1.017	-0.36	0.887
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.938	1.013	0.50	0.939	1.055	0.45	0.922
(c) $\epsilon \sim \text{Extreme Type I}$									
$E(Y X_1 = 0, X_2 = 0)$	-0.58	-0.58	0.950	0.997	-0.53	0.933	1.000	-0.58	0.931
$E(Y X_1 = 1, X_2 = 0)$	0.42	0.47	0.896	1.032	0.49	0.880	0.967	0.44	0.929
$E(Y X_1 = 0, X_2 = 1)$	-1.08	-1.18	0.953	0.744	-1.11	0.948	0.857	-1.10	0.930
$E(Y X_1 = 1, X_2 = 1)$	-0.08	-0.04	0.922	1.030	0.00	0.908	1.009	-0.07	0.933
(d) $\epsilon \sim \text{Extreme Type II}$									
$E(Y X_1 = 0, X_2 = 0)$	0.58	0.53	0.893	1.022	0.50	0.878	0.949	0.61	0.879
$E(Y X_1 = 1, X_2 = 0)$	1.58	1.58	0.945	0.997	1.53	0.930	0.996	1.44	0.899
$E(Y X_1 = 0, X_2 = 1)$	0.08	0.07	0.896	1.193	0.06	0.891	1.122	0.27	0.817
$E(Y X_1 = 1, X_2 = 1)$	1.08	1.03	0.925	1.014	1.00	0.908	0.979	0.99	0.881

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.7. The performance of CPMs on estimating conditional means with the sample size of 50 (continued). The results are based on 10,000 simulation replicates.

	true	probit			logit			cloglog			loglog		
		est	CP	RE	est	CP	RE	est	CP	RE	est	CP	RE
(e) $\epsilon \sim t$ with df=5													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.01	0.937	1.054	-0.01	0.938	1.123	0.02	0.905	0.979	0.09	0.925	0.902
$E(Y X_1 = 1, X_2 = 0)$	1	1.01	0.942	1.058	1.01	0.940	1.133	0.91	0.931	0.919	0.98	0.905	0.973
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.52	0.938	0.935	-0.52	0.939	1.024	-0.37	0.855	0.828	-0.43	0.921	0.720
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.936	1.071	0.50	0.936	1.145	0.44	0.910	0.994	0.54	0.911	0.991
(f) $\epsilon \sim \text{uniform}(-5, 5)$													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.08	0.927	0.962	0.00	0.924	0.913	-0.03	0.928	1.132	-0.04	0.926	0.964
$E(Y X_1 = 1, X_2 = 0)$	1	1.07	0.926	0.959	0.99	0.925	0.910	1.03	0.924	0.958	1.02	0.929	1.132
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.65	0.912	0.960	-0.49	0.917	0.913	-0.54	0.925	1.200	-0.55	0.920	0.974
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.49	0.930	0.984	0.48	0.926	0.920	0.50	0.931	1.073	0.49	0.928	1.071
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$													
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.948	0.968	0.04	0.932	0.888	0.00	0.934	1.133	0.11	0.910	0.674
$E(Y X_1 = 1, X_2 = 0)$	1	1.03	0.905	0.940	1.05	0.894	0.841	1.01	0.932	1.023	0.97	0.891	0.842
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.55	0.943	0.840	-0.49	0.929	0.836	-0.50	0.926	1.080	-0.39	0.905	0.597
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.53	0.928	0.945	0.55	0.914	0.864	0.50	0.934	1.148	0.55	0.902	0.789
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.03	0.912	0.943	-0.04	0.895	0.838	0.03	0.893	0.848	-0.01	0.930	1.010
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.946	0.973	0.96	0.929	0.887	0.89	0.907	0.662	1.00	0.931	1.127
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.49	0.901	1.069	-0.49	0.894	0.942	-0.33	0.808	0.654	-0.50	0.922	1.098
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.46	0.929	0.946	0.44	0.910	0.858	0.42	0.890	0.769	0.49	0.934	1.147

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.8. The performance of CPMs on estimating conditional medians with the sample size of 50.

	true	probit		logit		cloglog		loglog					
		est	CP	est	CP	RE	est	CP	RE	est	CP	RE	
(a) $\epsilon \sim \text{Normal}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.03	0.945	1.148	0.02	0.946	1.140	0.11	0.923	0.957	0.03	0.948	0.996
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.04	0.943	1.095	1.04	0.943	1.069	1.04	0.953	0.940	0.95	0.935	1.089
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.47	0.945	1.265	-0.48	0.947	1.255	-0.30	0.878	0.853	-0.49	0.947	0.976
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.53	0.946	1.214	0.52	0.946	1.158	0.55	0.936	1.123	0.49	0.938	1.157
(b) $\epsilon \sim \text{Logistic}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.07	0.942	1.088	0.04	0.946	1.095	0.17	0.919	0.972	0.11	0.932	0.969
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.02	0.948	1.115	1.04	0.948	1.088	0.98	0.948	1.046	0.92	0.937	1.110
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.40	0.942	1.171	-0.45	0.948	1.201	-0.21	0.882	0.924	-0.36	0.931	0.935
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.54	0.945	1.226	0.54	0.948	1.172	0.56	0.942	1.233	0.51	0.940	1.222
(c) $\epsilon \sim \text{Extreme Type I}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.37	-0.39	0.956	1.090	-0.37	0.956	1.095	-0.34	0.946	1.251	-0.30	0.941	0.866
$Q^{0.5} X_1 = 1, X_2 = 0$	0.63	0.61	0.933	1.207	0.65	0.936	1.183	0.68	0.947	1.168	0.47	0.894	0.832
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.87	-0.96	0.956	0.939	-0.95	0.954	0.972	-0.86	0.947	1.300	-0.79	0.939	0.689
$Q^{0.5} X_1 = 1, X_2 = 1$	0.13	0.12	0.948	1.282	0.15	0.948	1.223	0.17	0.946	1.389	0.09	0.937	1.097
(d) $\epsilon \sim \text{Extreme Type II}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0.37	0.46	0.921	1.108	0.42	0.931	1.159	0.60	0.854	0.681	0.38	0.945	1.222
$Q^{0.5} X_1 = 1, X_2 = 0$	1.37	1.46	0.950	0.941	1.45	0.950	0.967	1.37	0.954	0.877	1.41	0.939	1.163
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.13	0.02	0.892	1.166	-0.02	0.918	1.276	0.27	0.692	0.504	-0.11	0.941	1.434
$Q^{0.5} X_1 = 1, X_2 = 1$	0.87	0.94	0.944	1.195	0.91	0.946	1.182	0.95	0.938	1.044	0.89	0.944	1.350

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.9. The performance of CPMs on estimating conditional medians with the sample size of 50 (continued).

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with df=5									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.06	0.939	0.985	0.04	0.946	1.050	0.15	0.909
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.01	0.949	1.034	1.03	0.953	1.044	0.98	0.904
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.41	0.939	1.002	-0.45	0.950	1.107	-0.23	0.848
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.53	0.948	1.134	0.53	0.950	1.124	0.54	0.939
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	-0.18	0.938	1.124	-0.09	0.942	1.065	-0.02	0.942
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.36	0.923	1.006	1.28	0.933	0.996	1.39	0.936
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.91	0.919	1.204	-0.75	0.934	1.177	-0.64	0.936
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.58	0.946	1.253	0.57	0.945	1.134	0.68	0.948
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0.13	0.09	0.952	1.146	0.11	0.951	1.137	0.15	0.945
$Q^{0.5} X_1 = 1, X_2 = 0$	1.13	1.14	0.938	1.317	1.16	0.932	1.214	1.21	0.942
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.37	-0.49	0.942	0.994	-0.46	0.947	1.038	-0.37	0.952
$Q^{0.5} X_1 = 1, X_2 = 1$	0.63	0.63	0.945	1.367	0.65	0.943	1.250	0.69	0.946
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.13	-0.07	0.927	1.251	-0.09	0.933	1.234	0.05	0.876
$Q^{0.5} X_1 = 1, X_2 = 0$	0.87	0.98	0.940	0.923	0.96	0.944	0.963	0.93	0.958
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.63	-0.52	0.904	1.397	-0.55	0.917	1.392	-0.31	0.750
$Q^{0.5} X_1 = 1, X_2 = 1$	0.37	0.43	0.940	1.294	0.40	0.940	1.237	0.45	0.935

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.10. The performance of CPMs on estimating conditional means with the sample size of 100. Part of these results are also summarized in Figure 11.

	true	probit		logit		cologlog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.941	0.982	0.00	0.940	0.955	0.02	0.930
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.941	0.982	0.99	0.938	0.948	0.91	0.669
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.50	0.944	0.973	-0.48	0.935	0.926	-0.37	0.818
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.939	0.974	0.49	0.936	0.951	0.44	0.914
(b) $\epsilon \sim \text{Logistic}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.945	1.016	0.00	0.945	1.054	0.05	0.918
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.944	1.015	1.00	0.944	1.054	0.90	0.928
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.51	0.945	0.984	-0.50	0.944	1.026	-0.34	0.869
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.942	1.018	0.50	0.944	1.070	0.45	0.821
(c) $\epsilon \sim \text{Extreme Type I}$									
$E(Y X_1 = 0, X_2 = 0)$	-0.58	-0.58	0.955	0.999	-0.53	0.932	0.969	-0.58	0.942
$E(Y X_1 = 1, X_2 = 0)$	0.42	0.47	0.901	0.989	0.50	0.879	0.881	0.43	0.942
$E(Y X_1 = 0, X_2 = 1)$	-1.08	-1.19	0.958	0.665	-1.11	0.955	0.860	-1.09	0.943
$E(Y X_1 = 1, X_2 = 1)$	-0.08	-0.03	0.928	1.000	0.00	0.910	0.938	-0.07	0.941
(d) $\epsilon \sim \text{Extreme Type II}$									
$E(Y X_1 = 0, X_2 = 0)$	0.58	0.53	0.896	0.990	0.50	0.878	0.884	0.61	0.892
$E(Y X_1 = 1, X_2 = 0)$	1.58	1.58	0.955	0.997	1.53	0.933	0.971	1.44	0.881
$E(Y X_1 = 0, X_2 = 1)$	0.08	0.07	0.910	1.194	0.05	0.903	1.106	0.28	0.739
$E(Y X_1 = 1, X_2 = 1)$	1.08	1.03	0.928	1.008	0.99	0.908	0.931	0.99	0.886

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.11. The performance of CPMs on estimating conditional means with the sample size of 100 (continued). Part of these results are also summarized in Figure 11.

	true	probit			logit			cloglog			loglog		
		est	CP	RE	est	CP	RE	est	CP	RE	est	CP	RE
(e) $\epsilon \sim t$ with df=5													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.01	0.942	1.060	-0.01	0.942	1.128	0.03	0.905	0.957	0.10	0.913	0.781
$E(Y X_1 = 1, X_2 = 0)$	1	1.01	0.944	1.065	1.01	0.944	1.141	0.89	0.911	0.755	0.97	0.911	0.957
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.53	0.945	0.918	-0.53	0.947	1.028	-0.35	0.806	0.676	-0.43	0.925	0.656
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.942	1.088	0.50	0.944	1.177	0.43	0.909	0.961	0.55	0.917	0.973
(f) $\epsilon \sim \text{uniform}(-5, 5)$													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.08	0.935	0.957	0.01	0.938	0.922	-0.03	0.941	1.183	-0.03	0.939	0.994
$E(Y X_1 = 1, X_2 = 0)$	1	1.09	0.935	0.952	0.99	0.936	0.919	1.04	0.940	0.994	1.03	0.943	1.170
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.66	0.922	0.912	-0.47	0.928	0.910	-0.54	0.940	1.255	-0.56	0.929	0.984
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.940	1.000	0.49	0.936	0.926	0.51	0.943	1.131	0.50	0.941	1.136
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$													
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.954	0.984	0.04	0.929	0.849	0.00	0.939	1.158	0.12	0.877	0.521
$E(Y X_1 = 1, X_2 = 0)$	1	1.03	0.908	0.916	1.05	0.891	0.792	1.01	0.940	1.024	0.97	0.898	0.840
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.56	0.945	0.812	-0.49	0.939	0.835	-0.49	0.932	1.106	-0.38	0.897	0.511
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.53	0.932	0.935	0.56	0.911	0.822	0.50	0.943	1.184	0.56	0.894	0.759
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$													
$E(Y X_1 = 0, X_2 = 0)$	0	-0.03	0.908	0.921	-0.05	0.895	0.795	0.03	0.896	0.832	0.00	0.936	1.018
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.955	0.985	0.95	0.925	0.841	0.87	0.873	0.509	1.00	0.943	1.166
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.49	0.909	1.072	-0.50	0.901	0.938	-0.32	0.712	0.475	-0.50	0.936	1.112
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.46	0.928	0.933	0.43	0.904	0.798	0.42	0.885	0.720	0.49	0.941	1.205

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.12. The performance of CPMs on estimating conditional medians with the sample size of 100. Part of these results are also summarized in Figure 12.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.01	0.951	1.130	0.00	0.949	1.108	0.10	0.908
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.02	0.944	1.138	1.03	0.940	1.092	1.00	0.952
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.49	0.945	1.226	-0.50	0.945	1.215	-0.30	0.809
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.51	0.948	1.236	0.51	0.947	1.169	0.52	0.940
(b) $\epsilon \sim \text{Logistic}$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.05	0.945	1.092	0.02	0.949	1.112	0.15	0.909
$Q^{0.5} X_1 = 1, X_2 = 0$	1	0.99	0.949	1.118	1.02	0.948	1.094	0.93	0.942
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.42	0.941	1.121	-0.48	0.949	1.176	-0.21	0.820
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.52	0.943	1.216	0.52	0.947	1.165	0.52	0.938
(c) $\epsilon \sim \text{Extreme Type I}$									
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.37	-0.40	0.958	1.068	-0.38	0.957	1.080	-0.35	0.950
$Q^{0.5} X_1 = 1, X_2 = 0$	0.63	0.59	0.931	1.163	0.64	0.936	1.184	0.66	0.948
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.87	-0.98	0.952	0.814	-0.97	0.949	0.847	-0.86	0.953
$Q^{0.5} X_1 = 1, X_2 = 1$	0.13	0.12	0.947	1.262	0.14	0.945	1.206	0.15	0.943
(d) $\epsilon \sim \text{Extreme Type II}$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0.37	0.44	0.920	1.054	0.40	0.934	1.156	0.58	0.782
$Q^{0.5} X_1 = 1, X_2 = 0$	1.37	1.44	0.950	0.944	1.42	0.951	0.978	1.32	0.948
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.13	0.00	0.873	1.003	-0.05	0.909	1.185	0.27	0.465
$Q^{0.5} X_1 = 1, X_2 = 1$	0.87	0.91	0.942	1.196	0.88	0.942	1.186	0.92	0.934

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.13. The performance of CPMs on estimating conditional medians with the sample size of 100 (continued). Part of these results are also summarized in Figure 12.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with df=5									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.05	0.941	0.964	0.02	0.951	1.046	0.13	0.882
$Q^{0.5} X_1 = 1, X_2 = 0$	1	0.98	0.949	1.032	1.01	0.952	1.057	0.93	0.934
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.42	0.933	0.955	-0.48	0.948	1.109	-0.23	0.725
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.52	0.950	1.127	0.52	0.951	1.124	0.51	0.942
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	-0.24	0.928	1.075	-0.14	0.940	1.068	-0.08	0.941
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.34	0.912	0.961	1.24	0.928	0.991	1.35	0.925
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.99	0.882	0.960	-0.81	0.920	1.074	-0.72	0.927
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.54	0.945	1.298	0.54	0.943	1.154	0.62	0.946
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0.13	0.08	0.948	1.099	0.10	0.949	1.122	0.14	0.948
$Q^{0.5} X_1 = 1, X_2 = 0$	1.13	1.12	0.937	1.318	1.15	0.932	1.216	1.20	0.933
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.37	-0.51	0.918	0.819	-0.47	0.925	0.904	-0.38	0.944
$Q^{0.5} X_1 = 1, X_2 = 1$	0.63	0.62	0.944	1.387	0.64	0.939	1.272	0.67	0.939
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.13	-0.09	0.924	1.231	-0.12	0.929	1.225	0.04	0.818
$Q^{0.5} X_1 = 1, X_2 = 0$	0.87	0.95	0.934	0.940	0.93	0.939	1.001	0.88	0.959
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.63	-0.54	0.898	1.292	-0.58	0.913	1.342	-0.31	0.550
$Q^{0.5} X_1 = 1, X_2 = 1$	0.37	0.40	0.945	1.347	0.38	0.940	1.274	0.42	0.936

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.14. The performance of CPMs on estimating conditional means with the sample size of 200.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.945	0.992	0.01	0.940	0.957	0.03	0.903
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.950	0.989	1.00	0.945	0.960	0.90	0.857
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.50	0.944	0.980	-0.48	0.934	0.920	-0.36	0.490
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.944	0.981	0.50	0.943	0.939	0.44	0.898
(b) $\epsilon \sim \text{Logistic}$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.948	1.018	0.00	0.949	1.063	0.05	0.914
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.946	1.020	1.00	0.947	1.057	0.89	0.910
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.51	0.948	0.985	-0.50	0.948	1.042	-0.32	0.806
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.947	1.022	0.50	0.947	1.077	0.45	0.927
(c) $\epsilon \sim \text{Extreme Type I}$									
$E(Y X_1 = 0, X_2 = 0)$	-0.58	-0.58	0.961	1.005	-0.53	0.935	0.909	-0.58	0.944
$E(Y X_1 = 1, X_2 = 0)$	0.42	0.47	0.887	0.930	0.50	0.853	0.764	0.43	0.945
$E(Y X_1 = 0, X_2 = 1)$	-1.08	-1.20	0.947	0.550	-1.11	0.961	0.862	-1.08	0.945
$E(Y X_1 = 1, X_2 = 1)$	-0.08	-0.03	0.928	0.966	0.00	0.897	0.836	-0.08	0.947
(d) $\epsilon \sim \text{Extreme Type II}$									
$E(Y X_1 = 0, X_2 = 0)$	0.58	0.53	0.891	0.932	0.50	0.856	0.769	0.62	0.880
$E(Y X_1 = 1, X_2 = 0)$	1.58	1.58	0.960	1.007	1.53	0.929	0.910	1.43	0.833
$E(Y X_1 = 0, X_2 = 1)$	0.08	0.07	0.910	1.198	0.05	0.901	1.097	0.30	0.552
$E(Y X_1 = 1, X_2 = 1)$	1.08	1.03	0.923	0.961	0.99	0.890	0.849	0.99	0.871

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.15. The performance of CPMs on estimating conditional means with the sample size of 200 (continued).

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with df=5									
$E(Y X_1 = 0, X_2 = 0)$	0	-0.01	0.948	1.060	-0.01	0.947	1.133	0.03	0.912
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.951	1.072	1.01	0.950	1.144	0.88	0.860
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.53	0.949	0.908	-0.52	0.949	1.034	-0.33	0.715
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.50	0.948	1.095	0.50	0.949	1.179	0.43	0.902
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$E(Y X_1 = 0, X_2 = 0)$	0	-0.08	0.934	0.928	0.02	0.943	0.923	-0.02	0.950
$E(Y X_1 = 1, X_2 = 0)$	1	1.09	0.934	0.922	0.99	0.939	0.920	1.03	0.941
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.67	0.910	0.822	-0.47	0.934	0.903	-0.54	0.942
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.49	0.948	1.014	0.49	0.941	0.931	0.50	0.950
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$E(Y X_1 = 0, X_2 = 0)$	0	0.00	0.961	0.989	0.05	0.921	0.778	0.00	0.948
$E(Y X_1 = 1, X_2 = 0)$	1	1.03	0.910	0.892	1.05	0.887	0.733	1.00	0.944
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.56	0.943	0.734	-0.49	0.948	0.831	-0.49	0.940
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.53	0.934	0.929	0.55	0.910	0.765	0.49	0.948
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$E(Y X_1 = 0, X_2 = 0)$	0	-0.03	0.910	0.888	-0.05	0.889	0.734	0.03	0.898
$E(Y X_1 = 1, X_2 = 0)$	1	1.00	0.960	0.989	0.96	0.923	0.781	0.87	0.807
$E(Y X_1 = 0, X_2 = 1)$	-0.5	-0.49	0.922	1.080	-0.50	0.915	0.938	-0.31	0.533
$E(Y X_1 = 1, X_2 = 1)$	0.5	0.47	0.932	0.914	0.44	0.900	0.723	0.42	0.878

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.16. The performance of CPMs on estimating conditional medians with the sample size of 200.

	true	probit		logit		cloglog		loglog					
		est	CP	est	CP	RE	est	CP	RE	est	CP	RE	
(a) $\epsilon \sim \text{Normal}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.01	0.947	1.123	0.00	0.944	1.110	0.09	0.868	0.706	0.04	0.938	0.884
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.01	0.948	1.114	1.02	0.944	1.065	0.98	0.952	0.942	0.92	0.896	0.807
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.49	0.949	1.200	-0.51	0.944	1.173	-0.30	0.652	0.392	-0.47	0.946	0.857
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.51	0.948	1.236	0.50	0.945	1.165	0.51	0.944	1.191	0.49	0.942	1.198
(b) $\epsilon \sim \text{Logistic}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.05	0.945	1.043	0.01	0.953	1.089	0.15	0.871	0.722	0.12	0.902	0.765
$Q^{0.5} X_1 = 1, X_2 = 0$	1	0.98	0.949	1.102	1.01	0.948	1.089	0.90	0.920	0.870	0.88	0.895	0.837
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.42	0.934	1.046	-0.49	0.949	1.173	-0.21	0.690	0.452	-0.31	0.868	0.620
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.51	0.947	1.206	0.51	0.949	1.153	0.51	0.945	1.259	0.51	0.939	1.258
(c) $\epsilon \sim \text{Extreme Type I}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.37	-0.40	0.956	1.024	-0.39	0.954	1.053	-0.36	0.948	1.249	-0.27	0.905	0.659
$Q^{0.5} X_1 = 1, X_2 = 0$	0.63	0.59	0.921	1.049	0.63	0.935	1.157	0.65	0.949	1.155	0.44	0.671	0.377
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.87	-0.98	0.932	0.703	-0.97	0.927	0.732	-0.86	0.949	1.256	-0.72	0.881	0.475
$Q^{0.5} X_1 = 1, X_2 = 1$	0.13	0.11	0.946	1.218	0.14	0.946	1.179	0.14	0.948	1.371	0.11	0.938	1.114
(d) $\epsilon \sim \text{Extreme Type II}$													
$Q^{0.5} X_1 = 0, X_2 = 0$	0.37	0.43	0.906	0.939	0.39	0.932	1.134	0.58	0.620	0.333	0.37	0.947	1.177
$Q^{0.5} X_1 = 1, X_2 = 0$	1.37	1.42	0.946	0.923	1.41	0.948	0.970	1.29	0.924	0.741	1.38	0.947	1.237
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.13	0.00	0.824	0.795	-0.06	0.900	1.109	0.28	0.178	0.160	-0.13	0.948	1.342
$Q^{0.5} X_1 = 1, X_2 = 1$	0.87	0.90	0.942	1.186	0.87	0.946	1.187	0.90	0.937	1.098	0.87	0.947	1.402

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.17. The performance of CPMs on estimating conditional medians with the sample size of 200 (continued).

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with df=5									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	0.04	0.938	0.910	0.01	0.949	1.030	0.13	0.822
$Q^{0.5} X_1 = 1, X_2 = 0$	1	0.97	0.945	0.985	1.00	0.951	1.065	0.90	0.888
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-0.42	0.926	0.847	-0.48	0.949	1.080	-0.22	0.512
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.51	0.950	1.146	0.51	0.950	1.139	0.50	0.941
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0	-0.26	0.906	0.937	-0.16	0.931	1.025	-0.11	0.938
$Q^{0.5} X_1 = 1, X_2 = 0$	1	1.32	0.884	0.829	1.22	0.919	0.934	1.31	0.904
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.5	-1.03	0.813	0.689	-0.84	0.898	0.922	-0.75	0.906
$Q^{0.5} X_1 = 1, X_2 = 1$	0.5	0.52	0.950	1.330	0.52	0.948	1.167	0.58	0.953
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	0.13	0.07	0.935	0.954	0.10	0.944	1.055	0.14	0.952
$Q^{0.5} X_1 = 1, X_2 = 0$	1.13	1.11	0.935	1.298	1.14	0.934	1.236	1.19	0.927
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.37	-0.52	0.866	0.585	-0.48	0.892	0.707	-0.38	0.948
$Q^{0.5} X_1 = 1, X_2 = 1$	0.63	0.61	0.947	1.345	0.63	0.945	1.250	0.66	0.948
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$Q^{0.5} X_1 = 0, X_2 = 0$	-0.13	-0.10	0.925	1.190	-0.13	0.932	1.216	0.03	0.699
$Q^{0.5} X_1 = 1, X_2 = 0$	0.87	0.95	0.918	0.829	0.92	0.930	0.937	0.87	0.956
$Q^{0.5} X_1 = 0, X_2 = 1$	-0.63	-0.55	0.884	1.155	-0.59	0.911	1.304	-0.31	0.276
$Q^{0.5} X_1 = 1, X_2 = 1$	0.37	0.40	0.943	1.321	0.37	0.940	1.262	0.41	0.937

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.18. The performance of CPMs on estimating the regression coefficients under more extreme covariates distributions and an interaction term with properly specified link functions.

	true	$\epsilon \sim N(0,1)$				$\epsilon \sim \text{Extreme}(0, 1)$			
		est	est.se	emp.se	CP	est	est.se	emp.se	CP
<i>n</i> = 25									
β_1	1.000	1.25	0.817	1.292	0.905	1.61	0.902	2.615	0.874
β_2	1.000	1.26	0.533	0.917	0.897	1.42	0.575	12.958	0.876
β_3	0.500	0.62	0.528	0.910	0.899	0.52	0.578	12.961	0.878
$\alpha(y_1)$	-2.707	-3.31	1.072	1.512	0.909	-3.09	1.159	2.705	0.907
$\alpha(y_2)$	-1.327	-1.64	0.904	1.359	0.920	-1.32	0.961	2.605	0.898
$\alpha(y_3)$	0.729	0.92	0.852	1.292	0.921	1.30	0.908	2.609	0.889
$\alpha(y_4)$	2.865	3.61	1.110	1.587	0.900	4.09	1.253	2.850	0.866
$\alpha(y_5)$	4.464	5.51	1.357	1.851	0.894	6.05	1.576	3.163	0.870
<i>n</i> = 50									
β_1	1.000	1.10	0.435	0.500	0.928	1.24	0.467	0.595	0.900
β_2	1.000	1.10	0.264	0.338	0.921	1.12	0.290	0.367	0.913
β_3	0.500	0.55	0.261	0.338	0.923	0.57	0.285	0.361	0.917
$\alpha(y_1)$	-2.707	-2.99	0.593	0.657	0.929	-2.92	0.646	0.757	0.929
$\alpha(y_2)$	-1.327	-1.45	0.489	0.546	0.934	-1.37	0.528	0.642	0.929
$\alpha(y_3)$	0.729	0.81	0.461	0.524	0.934	0.93	0.473	0.590	0.913
$\alpha(y_4)$	2.865	3.15	0.602	0.676	0.927	3.36	0.649	0.782	0.894
$\alpha(y_5)$	4.464	4.92	0.761	0.843	0.920	5.16	0.856	0.995	0.892
<i>n</i> = 100									
β_1	1.000	1.05	0.281	0.296	0.937	1.10	0.293	0.319	0.922
β_2	1.000	1.04	0.166	0.176	0.935	1.06	0.177	0.190	0.933
β_3	0.500	0.53	0.164	0.177	0.936	0.54	0.173	0.186	0.934
$\alpha(y_1)$	-2.707	-2.83	0.383	0.405	0.938	-2.82	0.418	0.443	0.938
$\alpha(y_2)$	-1.327	-1.38	0.318	0.333	0.941	-1.35	0.341	0.365	0.935
$\alpha(y_3)$	0.729	0.76	0.299	0.313	0.944	0.81	0.300	0.321	0.929
$\alpha(y_4)$	2.865	3.00	0.390	0.411	0.940	3.09	0.401	0.431	0.926
$\alpha(y_5)$	4.464	4.68	0.494	0.519	0.932	4.79	0.537	0.569	0.915
<i>n</i> = 200									
β_1	1.000	1.02	0.192	0.195	0.948	1.05	0.198	0.203	0.941
β_2	1.000	1.02	0.112	0.114	0.945	1.02	0.118	0.123	0.940
β_3	0.500	0.51	0.111	0.115	0.944	0.51	0.115	0.120	0.939
$\alpha(y_1)$	-2.707	-2.76	0.261	0.268	0.946	-2.75	0.285	0.295	0.943
$\alpha(y_2)$	-1.327	-1.35	0.218	0.223	0.946	-1.33	0.233	0.239	0.948
$\alpha(y_3)$	0.729	0.74	0.205	0.208	0.950	0.77	0.204	0.210	0.945
$\alpha(y_4)$	2.865	2.93	0.266	0.273	0.944	2.97	0.267	0.276	0.935
$\alpha(y_5)$	4.464	4.56	0.336	0.341	0.947	4.61	0.358	0.365	0.939
<i>n</i> = 500									
β_1	1.000	1.01	0.119	0.120	0.946	1.02	0.122	0.124	0.945
β_2	1.000	1.01	0.069	0.069	0.948	1.01	0.072	0.072	0.952
β_3	0.500	0.50	0.069	0.070	0.946	0.51	0.071	0.071	0.948
$\alpha(y_1)$	-2.707	-2.73	0.162	0.164	0.949	-2.73	0.177	0.178	0.946
$\alpha(y_2)$	-1.327	-1.34	0.136	0.138	0.948	-1.33	0.145	0.148	0.948
$\alpha(y_3)$	0.729	0.73	0.128	0.129	0.948	0.75	0.127	0.130	0.945
$\alpha(y_4)$	2.865	2.89	0.165	0.166	0.952	2.91	0.164	0.167	0.943
$\alpha(y_5)$	4.464	4.50	0.209	0.212	0.943	4.52	0.219	0.223	0.945
<i>n</i> = 1000									
β_1	1.000	1.00	0.084	0.084	0.949	1.01	0.085	0.087	0.944
β_2	1.000	1.00	0.049	0.049	0.945	1.01	0.051	0.051	0.946
β_3	0.500	0.50	0.048	0.049	0.946	0.50	0.049	0.050	0.942
$\alpha(y_1)$	-2.707	-2.72	0.114	0.116	0.948	-2.72	0.124	0.125	0.949
$\alpha(y_2)$	-1.327	-1.33	0.095	0.096	0.946	-1.33	0.102	0.102	0.951
$\alpha(y_3)$	0.729	0.73	0.090	0.090	0.950	0.74	0.089	0.091	0.944
$\alpha(y_4)$	2.865	2.88	0.116	0.116	0.950	2.89	0.115	0.116	0.944
$\alpha(y_5)$	4.464	4.48	0.147	0.147	0.948	4.49	0.153	0.154	0.944

est is the mean of the point estimates.

est.se is the mean of the standard error estimates.

emp.se is the standard deviation of the point estimates

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates

Table S.19. The performance of CPMs on estimating conditional means under more extreme covariates distributions and an interaction term with the sample size of 100.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	0.00	0.933	0.897	0.00	0.925	0.857	-0.10	0.882
$E(Y Z_1 = 1, Z_2 = 0)$	1	1.00	0.939	0.547	1.00	0.937	0.521	0.94	0.926
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.939	0.957	1.00	0.928	0.911	0.90	0.888
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.50	0.941	0.673	2.50	0.936	0.643	2.43	0.919
(b) $\epsilon \sim \text{Logistic}$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	0.00	0.937	0.986	0.00	0.933	1.038	-0.15	0.857
$E(Y Z_1 = 1, Z_2 = 0)$	1	1.00	0.941	0.881	1.00	0.938	0.905	0.81	0.874
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.939	0.996	1.00	0.937	1.052	0.81	0.886
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.50	0.946	0.968	2.50	0.941	1.009	2.25	0.873
(c) $\epsilon \sim \text{Extreme Type I}$									
$E(Y Z_1 = 0, Z_2 = 0)$	-0.58	-0.55	0.939	1.004	-0.48	0.906	0.933	-0.60	0.927
$E(Y Z_1 = 1, Z_2 = 0)$	0.42	0.48	0.919	0.621	0.56	0.846	0.426	0.45	0.933
$E(Y Z_1 = 0, Z_2 = 1)$	0.42	0.43	0.936	0.986	0.52	0.911	0.980	0.40	0.927
$E(Y Z_1 = 1, Z_2 = 1)$	1.92	1.98	0.912	0.708	2.06	0.846	0.523	1.95	0.933
(d) $\epsilon \sim \text{Extreme Type II}$									
$E(Y Z_1 = 0, Z_2 = 0)$	0.58	0.55	0.929	0.940	0.47	0.905	0.906	0.38	0.779
$E(Y Z_1 = 1, Z_2 = 0)$	1.58	1.52	0.924	0.635	1.44	0.850	0.444	1.33	0.774
$E(Y Z_1 = 0, Z_2 = 1)$	1.58	1.56	0.941	1.027	1.48	0.917	1.000	1.35	0.827
$E(Y Z_1 = 1, Z_2 = 1)$	3.08	3.03	0.941	0.873	2.95	0.875	0.616	2.78	0.768

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression measured with MSE ratios.

Table S.20. The performance of CPMs on estimating conditional means under more extreme covariates distributions and an interaction term with the sample size of 100 (continued). The results are based on 10,000 simulation replicates.

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with $df=5$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	0.00	0.942	0.998	0.00	0.941	1.109	-0.14	0.830
$E(Y Z_1 = 1, Z_2 = 0)$	1	1.00	0.942	0.710	1.00	0.943	0.767	0.85	0.868
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.939	1.011	1.00	0.938	1.135	0.84	0.862
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.50	0.945	0.873	2.50	0.946	0.943	2.32	0.879
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	-0.03	0.938	0.941	0.00	0.920	0.791	-0.15	0.924
$E(Y Z_1 = 1, Z_2 = 0)$	1	1.01	0.928	0.810	1.01	0.910	0.660	0.93	0.930
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.946	0.995	1.00	0.926	0.842	0.80	0.928
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.57	0.922	0.819	2.50	0.908	0.677	2.41	0.931
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	0.00	0.938	0.907	0.05	0.914	0.797	-0.03	0.932
$E(Y Z_1 = 1, Z_2 = 0)$	1	1.02	0.934	0.529	1.07	0.901	0.412	1.00	0.937
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.933	0.933	1.05	0.916	0.839	0.97	0.930
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.52	0.924	0.618	2.57	0.891	0.498	2.50	0.934
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$E(Y Z_1 = 0, Z_2 = 0)$	0	0.00	0.937	0.882	-0.05	0.917	0.789	-0.13	0.856
$E(Y Z_1 = 1, Z_2 = 0)$	1	0.99	0.937	0.538	0.94	0.902	0.425	0.89	0.893
$E(Y Z_1 = 0, Z_2 = 1)$	1	1.00	0.939	0.951	0.95	0.919	0.854	0.86	0.866
$E(Y Z_1 = 1, Z_2 = 1)$	2.5	2.49	0.941	0.706	2.43	0.905	0.551	2.37	0.892

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified linear regression, measured with MSE ratios.

Table S.21. The performance of CPMs on estimating conditional medians under more extreme covariates distributions and an interaction term with the sample size of 100.

	true	probit		logit		cloglog		log log	
		est	CP	RE	est	CP	RE	est	CP
(a) $\epsilon \sim \text{Normal}$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0	0.04	0.941	1.012	0.04	0.936	0.977	0.13	0.903
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1	1.05	0.946	0.415	1.05	0.944	0.402	1.17	0.890
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1	1.04	0.946	1.177	1.04	0.940	1.133	1.3	0.907
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.5	2.55	0.945	0.511	2.55	0.944	0.496	2.67	0.903
(b) $\epsilon \sim \text{Logistic}$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0	0.04	0.945	1.010	0.04	0.943	1.060	0.10	0.897
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1	1.04	0.950	0.569	1.05	0.947	0.581	1.12	0.947
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1	1.04	0.945	1.104	1.04	0.944	1.161	1.11	0.919
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.5	2.55	0.952	0.697	2.55	0.950	0.712	2.59	0.958
(c) $\epsilon \sim \text{Extreme Type I}$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	-0.37	-0.51	0.927	0.857	-0.43	0.940	1.007	-0.36	0.938
$Q^{0.5} Z_1 = 1, Z_2 = 0$	0.63	0.53	0.934	0.383	0.61	0.949	0.460	0.69	0.944
$Q^{0.5} Z_1 = 0, Z_2 = 1$	0.63	0.48	0.927	0.897	0.56	0.941	1.077	0.64	0.941
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.13	2.04	0.933	0.475	2.12	0.947	0.559	2.20	0.947
(d) $\epsilon \sim \text{Extreme Type II}$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0.37	0.59	0.898	0.683	0.51	0.925	0.852	0.59	0.851
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1.37	1.56	0.887	0.282	1.48	0.926	0.380	1.60	0.888
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1.37	1.61	0.908	0.804	1.52	0.932	1.000	1.62	0.876
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.87	3.07	0.893	0.356	2.99	0.933	0.486	3.09	0.915

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.22. The performance of CPMs on estimating conditional medians under more extreme covariates distributions and an interaction term with the sample size of 100 (continued).

	true	probit		logit		cloglog		loglog	
		est	CP	RE	est	CP	RE	est	CP
(e) $\epsilon \sim t$ with df=5									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0	0.03	0.951	0.853	0.04	0.949	0.930	0.09	0.895
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1	1.04	0.948	0.370	1.04	0.949	0.393	1.12	0.929
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1	1.04	0.948	0.945	1.04	0.948	1.041	1.11	0.905
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.5	2.55	0.951	0.455	2.55	0.950	0.482	2.61	0.934
(f) $\epsilon \sim \text{uniform}(-5, 5)$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0	0.01	0.944	1.764	0.01	0.931	1.467	0.18	0.935
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1	1.09	0.930	1.285	1.09	0.919	1.108	1.34	0.888
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1	1.08	0.947	1.966	1.08	0.933	1.634	1.20	0.940
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.5	2.70	0.921	1.281	2.67	0.916	1.150	2.92	0.890
(g) $\epsilon \sim \text{standardized Beta}(5, 2)$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	0.13	0.04	0.938	1.048	0.09	0.939	1.055	0.18	0.941
$Q^{0.5} Z_1 = 1, Z_2 = 0$	1.13	1.06	0.939	0.444	1.11	0.943	0.464	1.21	0.937
$Q^{0.5} Z_1 = 0, Z_2 = 1$	1.13	1.04	0.934	1.173	1.09	0.934	1.174	1.18	0.938
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.63	2.56	0.932	0.539	2.62	0.938	0.555	2.71	0.937
(h) $\epsilon \sim \text{standardized Beta}(2, 5)$									
$Q^{0.5} Z_1 = 0, Z_2 = 0$	-0.13	0.04	0.904	0.828	0.00	0.920	0.904	0.10	0.855
$Q^{0.5} Z_1 = 1, Z_2 = 0$	0.87	1.03	0.884	0.308	0.98	0.916	0.372	1.15	0.805
$Q^{0.5} Z_1 = 0, Z_2 = 1$	0.87	1.04	0.913	0.986	0.99	0.924	1.061	1.11	0.865
$Q^{0.5} Z_1 = 1, Z_2 = 1$	2.37	2.54	0.895	0.392	2.48	0.923	0.466	2.64	0.837

est is the mean of the point estimates.

CP is the coverage probability of 95% confidence intervals in the 10,000 simulation replicates.

RE is the relative efficiency compared with properly specified median regression measured with MSE ratios.

Table S.23. Average estimation time, average number of distinct outcomes, percent bias, and root mean squared error (rmSE) based on 100 simulation replications with $\epsilon \sim N(0, 1)$.

n	n distinct values	time (seconds)	$\beta_1 = 1$			$\beta_2 = -0.5$			$\alpha(y_1) = -1$			$\alpha(y_2) = -0.33$			$\alpha(y_3) = 0.5$			$\alpha(y_4) = 1.33$			$\alpha(y_5) = 2$		
			%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	%-bias	rmSE	
1000	1000	1.304	0.002	0.062	0.001	0.031	0.001	0.07	0.003	0.053	0.006	0.056	0.001	0.058	0.001	0.073	0.001	0.073	0.001	0.073			
1000	500	0.368	-0.008	0.065	-0.002	0.032	0.005	0.065	0.019	0.057	-0.006	0.052	-0.005	0.064	-0.003	0.076	-0.003	0.076	-0.015	0.082			
1000	100	0.084	-0.008	0.065	-0.002	0.032	0.031	0.074	0.069	0.062	-0.031	0.054	-0.018	0.066	-0.015	0.082	-0.015	0.082	-0.015	0.082			
5000	5000	20.823	0	0.032	0	0.016	-0.001	0.026	0.006	0.025	-0.003	0.025	0.025	0.025	0	0.032	-0.001	0.035	-0.004	0.034			
5000	1000	3.685	-0.003	0.028	-0.001	0.015	0.005	0.027	0.005	0.022	-0.009	0.024	-0.004	0.027	-0.004	0.034	-0.004	0.034	-0.006	0.035			
5000	500	1.484	-0.003	0.028	-0.001	0.015	0.008	0.028	0.011	0.022	-0.013	0.024	-0.006	0.028	-0.006	0.035	-0.006	0.035	-0.006	0.035			
5000	100	0.314	-0.003	0.027	-0.001	0.015	0.033	0.048	0.059	0.031	-0.039	0.031	-0.016	0.035	-0.017	0.051	-0.017	0.051	-0.017	0.051			
10000	10000	81.637	0.001	0.02	0.004	0.011	0.003	0.02	0.005	0.016	-0.002	0.017	0	0.018	0.001	0.024	0.001	0.024	0.001	0.024			
10000	5000	34.932	0.002	0.021	0.004	0.012	0.003	0.021	0.003	0.016	-0.001	0.017	-0.002	0.019	0	0.024	0.001	0.024	0.001	0.024			
10000	1000	5.831	0.002	0.021	0.004	0.012	0.006	0.022	0.008	0.016	-0.003	0.017	-0.004	0.02	-0.002	0.023	-0.002	0.023	-0.001	0.023			
10000	500	2.873	0.002	0.021	0.004	0.012	0.009	0.023	0.013	0.017	-0.007	0.017	-0.005	0.02	-0.002	0.024	-0.002	0.024	-0.001	0.024			
10000	100	0.602	0.002	0.021	0.004	0.011	0.033	0.046	0.067	0.031	-0.032	0.025	-0.016	0.031	-0.015	0.041	-0.015	0.041	-0.015	0.041			
20000	20000	338.117	0	0.016	0	0.007	0.001	0.013	-0.004	0.013	-0.002	0.013	0	0.013	0	0.016	0	0.016	0	0.016			
20000	10000	138.062	0	0.016	0	0.007	0.002	0.013	-0.003	0.013	-0.003	0.013	-0.001	0.014	-0.001	0.016	-0.001	0.016	-0.001	0.016			
20000	5000	61.745	0	0.016	0	0.007	0.002	0.013	-0.002	0.013	-0.003	0.013	-0.001	0.014	-0.001	0.016	-0.001	0.016	-0.001	0.016			
20000	1000	11.393	0	0.016	0	0.007	0.005	0.013	0.002	0.013	-0.006	0.013	-0.002	0.014	-0.002	0.017	-0.002	0.017	-0.002	0.017			
20000	500	5.668	0	0.016	0	0.007	0.008	0.016	0.008	0.013	-0.008	0.014	-0.003	0.014	-0.004	0.018	-0.003	0.018	-0.004	0.018			
20000	100	1.18	0	0.016	0	0.007	0.028	0.04	0.05	0.025	-0.036	0.024	-0.014	0.027	-0.017	0.045	-0.017	0.045	-0.017	0.045			
10^7	200	1477	0	0.001	0	0	0.008	0.006	0.006	0.002	-0.015	0.011	-0.013	0.017	-0.012	0.024	-0.012	0.024	-0.012	0.024			

^a 0 denotes < 0.001 .

Table S.24. Average estimation time, average number of distinct outcomes, percent bias, and root mean squared error (rMSE) based on 100 simulation replications with $\epsilon \sim \text{Type I extreme value distribution}$.

n	n distinct values	time (seconds)	$\beta_1 = 1$			$\beta_2 = -0.5$			$\alpha(y_1) = -1$			$\alpha(y_2) = -0.33$			$\alpha(y_3) = 0.5$			$\alpha(y_4) = 1.33$			$\alpha(y_5) = 2$		
			%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	%-bias	rMSE	
1000	1000	1.427	0.004	0.069	-0.001	0.036	0.001	0.072	-0.021	0.059	0.017	0.053	0.003	0.052	0	0.079							
1000	500	0.447	0.018	0.076	0.011	0.033	0.006	0.069	0.013	0.058	0.009	0.058	0.002	0.068	0.01	0.083							
1000	100	0.105	0.017	0.076	0.009	0.033	0.026	0.076	0.06	0.063	-0.021	0.059	-0.011	0.071	-0.012	0.086							
5000	5000	24.404	-0.005	0.028	-0.005	0.014	0.006	0.032	0.004	0.026	-0.005	0.023	-0.004	0.028	-0.002	0.037							
5000	1000	3.868	-0.006	0.03	0.004	0.016	0.005	0.031	0.014	0.025	-0.008	0.023	-0.004	0.029	-0.005	0.034							
5000	500	1.863	-0.007	0.03	0.004	0.016	0.008	0.031	0.02	0.025	-0.011	0.024	-0.007	0.029	-0.007	0.036							
5000	100	0.402	-0.007	0.03	0.003	0.016	0.029	0.046	0.065	0.035	-0.036	0.03	-0.02	0.04	-0.029	0.07							
10000	10000	95.653	-0.003	0.025	-0.001	0.011	0.004	0.024	0.003	0.019	-0.002	0.019	-0.002	0.02	-0.001	0.025							
10000	5000	42.273	-0.003	0.024	-0.001	0.011	0.004	0.025	0.007	0.02	-0.005	0.018	-0.005	0.019	-0.001	0.024							
10000	1000	7.248	-0.003	0.024	-0.001	0.011	0.006	0.025	0.012	0.02	-0.007	0.018	-0.007	0.019	-0.003	0.025							
10000	500	3.597	-0.003	0.024	-0.001	0.011	0.009	0.026	0.018	0.021	-0.01	0.019	-0.004	0.019	-0.005	0.026							
10000	100	0.77	-0.003	0.024	-0.001	0.01	0.028	0.04	0.067	0.03	-0.038	0.028	-0.028	0.034	-0.029	0.068							
20000	20000	393.904	-0.001	0.014	0.003	0.008	0	0.015	-0.003	0.013	0.003	0.01	0.013	0	0.016								
20000	10000	166.386	-0.001	0.014	0.003	0.008	0.001	0.015	-0.002	0.013	0.002	0.01	-0.001	0.013	0	0.016							
20000	5000	75.718	-0.001	0.014	0.003	0.008	0.001	0.015	-0.001	0.013	0.002	0.01	-0.001	0.013	-0.001	0.016							
20000	1000	14.224	-0.001	0.014	0.003	0.008	0.003	0.015	0.003	0.013	-0.001	0.013	-0.002	0.013	-0.002	0.017							
20000	500	7.112	-0.001	0.014	0.002	0.008	0.006	0.016	0.01	0.013	-0.004	0.01	-0.004	0.014	-0.004	0.019							
20000	100	1.514	-0.001	0.014	0.003	0.008	0.023	0.031	0.061	0.026	-0.037	0.022	-0.016	0.027	-0.032	0.068							

a 0 denotes < 0.001 .

Table S.25. Descriptive statistics of variables in the application example

	n=4776
Age	
Mean (SD)	36.4 (10.19)
Median (Range)	35 (18 - 82)
Gender	
Female	1179 (24.7%)
Male	3597 (75.3%)
Treatment Class	
Boosted PI (BPI)	655 (13.7%)
NNRTI	3888 (81.4%)
Unboosted PI (UBPI)	145 (3.0%)
Other	88 (1.8%)
Site	
Argentina	956 (20.0%)
Brazil	1326 (27.8%)
Chile	641 (13.4%)
Honduras	47 (1.0%)
Mexico	644 (13.5%)
Peru	1162 (24.3%)
Probable Infection Route	
Heterosexual	2089 (43.7%)
Homo/bisexual	2109 (44.2%)
IDU	52 (1.1%)
Other	34 (0.7%)
Unknown	492 (10.3%)
Year of ART Initiation	
Mean (SD)	2007.9 (3.4)
Median (Range)	2008 (2000 - 2013)
Baseline Nadir CD4	
Mean (SD)	176 (129)
Median (Range)	167 (0 - 934)
Baseline Viral Load	
Mean (SD)	297684 (1060601)
Median (Range)	91728 (19 - 3.3e+07)
Baseline AIDS status	
AIDS	1142 (23.9%)
not AIDS	2826 (59.2%)
Unknown	808 (16.9%)
6-month CD4	
Mean (SD)	330 (193)
Median (Range)	302 (1 - 2010)
6-month Viral Load	
Undetectable	2532 (85.5%)
Detectable	2244 (14.5%)
Mean (SD)	60624 (353575)
Median (Range)	1300 (400 - 7800000)

Table S.26. Type I error rate and power of CPMs for outcomes subject to detection limit (DL) compared with logistic regression models (dichotomizing the outcome into two categories: detectable and undetectable) and linear regression models (imputing measures below DL as equal to DL or 0). The sample size is 100 and we repeat the simulation 10,000 times for each scenario.

% below DL	orm		logistic regression binary outcome	linear regression	
	probit	logit		impute DL	impute 0
10%	H_0	0.053	0.053	0.042	0.050
	H_1	0.689	0.671	0.263	0.674
25%	H_0	0.052	0.053	0.045	0.050
	H_1	0.673	0.654	0.406	0.646
50%	H_0	0.052	0.049	0.044	0.049
	H_1	0.606	0.586	0.478	0.549
75%	H_0	0.051	0.049	0.045	0.049
	H_1	0.464	0.446	0.412	0.387
90%	H_0	0.042	0.041	0.041	0.050
	H_1	0.279	0.271	0.261	0.237